

Some Macroeconomic Aspects of Global Population Aging

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Key words: population aging, macroeconomic, demographic transition, consequences, human capital, economic growth, support ratio, demand for wealth

Research for this paper was funded by parallel grants from the National Institutes of Health to Lee and Mason, NIA R37 AG025247 and R01 AG025488, as well as by grants from MEXT.ACADEMIC FRONTIER (2006-2010) and UNFPA (RAS5P203) to NUPRI in Japan. During the final preparation of this paper Mason was Visiting Professor, Department of Global Health and Population and the Center for Population and Development Studies, Harvard University. We are grateful for help from Gretchen Donehower, Amonthep Chawla, and Miguel Romero, and grateful to all the country research teams in the NTA project for the use of their data. The researchers are identified and more detailed information is available for many countries in working papers on the NTA website: www.ntaccounts.org

Abstract

Across the demographic transition, declining mortality followed by declining fertility produces decades of rising support ratios as child dependency falls. These improving support ratios raise per capita consumption, other things equal, but eventually deteriorate as the population ages. Population aging and the forces leading to it can produce not only frightening declines in support ratios, but also very substantial increases in productivity and per capita income by raising investment in physical and human capital. Longer life, lower fertility, and population aging all raise the demand for wealth needed to provide for old age consumption. This leads to increased capital per worker even as aggregate saving rates fall. However, increased capital per worker may not occur if the increased demand for wealth is satisfied by increased familial or public pension transfers to the elderly. Thus institutions and policies matter for the consequences of population aging. The accumulation of human capital also varies across the transition. Lower fertility and mortality are associated with higher human capital investment per child, also raising labor productivity. Together, the positive changes due to human and physical capital accumulation will likely outweigh the problems of declining support ratios. We will draw on estimates and analyses from the National Transfer Accounts project to illustrate and quantify these points.

Introduction

Most countries around the world are experiencing low or declining fertility and mortality, and consequently population aging is in their future. No country in the world is yet close to experiencing full population aging; virtually all will see a doubling or tripling of their old age dependency ratios as conventionally defined, by the end of this century. This rise in projected dependency ratios suggests that in the future there will be fewer workers to support each retired elder, and therefore either that taxes and transfers will have to be substantially raised, or that workers will have to save more throughout their lives, or that labor supply will have to be extended into years which people currently expect to spend in retirement.

The prospect of population aging is a worrisome concern for policy makers, economists and the public. It threatens the sustainability of our public pension and health care systems, absent painful reforms. It raises the prevalence of every ill that afflicts the elderly, such as activity limitations, chronic care needs or dementia. It adds urgency to questions about the adequacy of the financial preparation of working generations for their retirement. Some analysts view population aging as economically catastrophic, and others view it as innocuous or advantageous, with most economists located someplace in between. These views of the whole carry over into its parts, such as saving adequacy, pension peril, and intergenerational conflict.

There are many topics in the macroeconomics of population aging that we could address, far more than space permits. Here we will focus on what we view to be the core issues: support ratios, capital intensity as it derives from desired wealth holdings, intergenerational transfers, and human capital. We will examine these issues in a cross-national context, in which questions about population aging are seen to be closely related to questions about the demographic transition and economic development.

Some Background on the Literature in these Areas

Population aging is driven by two engines in a closed population: slowing population growth rates due to falling fertility and longer life. Both of these fundamental causes operate across the demographic transition, and they have other consequences besides aging, complicating any treatment of this topic.

We will start with neoclassical growth models (Solow, 1956), which do not even include population age distribution, but do include population growth rates. These models show that if savings rates remain constant, slower population growth causes capital intensification, raising the productivity of labor and per capita consumption. If instead of holding savings rates constant we always choose the savings rate that maximizes steady state per capita consumption (the golden rule), then we find that slower population growth (and therefore population aging) causes savings rates to fall at the same time that capital intensity and per capita consumption rise. This simple fact is often overlooked, but we are forcefully reminded of it in Cutler et al (1990). Population aging is not generally a reason for policy to strive to raise national saving rates, although population aging may intensify the need to remedy saving rates that were too low to begin with.

Two confluent lines of the theoretical literature, unlike the neoclassical growth model, do explicitly incorporate population age distribution in an enlightening way. One line started with a seminal article by Samuelson (1958) which explored the theoretical importance of intergenerational transfers in the context of an overlapping generations model with no durable goods. He showed that transfers can act as a store of wealth and allow the economy to reach a more desirable steady state than could be attained through exchange in a competitive economy. Diamond (1965) added capital to this simple model, melding it with the Solow model. Samuelson (1976) suggested that an optimal population growth rate would optimize the tradeoff between capital dilution and the old age support ratio (although Deardorff, 1976, showed that Samuelson's optimum was actually a minimum, leading to a later reformulation by Samuelson), and in a comment on this article Arthur-McNicoll (1978) developed a more fully age-structured model and showed how to analyze the effects of variations in population growth rates across golden rule steady states. Willis, building on Gale (1973), developed a system of age accounting that elegantly showed the relation between capital and transfers as stores of wealth (Willis, 1988). Assuming altruistic dynastic utility he made fertility and intergenerational transfers endogenous within a general framework drawing on ideas from Becker and his collaborators (Willis, 1987). Lee (1994a and b) and Bommier and Lee (2003) further extended this framework, with an emphasis on the implications of patterns of intergenerational transfers through specific public and private channels.

Parallel to these developments, Becker and Barro (1988), Becker and Tomes (1976), Becker and Murphy (1988), and other studies developed a theory of fertility, investments in human capital, bequests to children, and public sector education and pensions, leading to a theory of economic growth. These can be viewed as decentralized age-structured Ramsey models of optimal economic growth.

While the work mentioned above is mainly theoretical, there are also important theory-based numerical simulations on which we elaborate. Tobin (1967) embedded realistic steady state demography in a neoclassical growth model with life cycle saving. Adults with children shared household consumption among household members in proportion to equivalent adult consumer (EAC) weights, and chose optimal consumption trajectories for themselves subject to life time budget constraints with annuitization of wealth and no bequests. The resulting age trajectories of assets by age were weighted by population and aggregated to find the aggregate demand for life cycle wealth, as a function of the interest rate, the rate of productivity growth, and the population growth rate. This aggregate demand for assets by households was also an aggregate supply of investment funds for producers. Tobin plotted this against the demand for capital by producers as a function of the interest rate, or rather actually, as a function of the interest rate minus the productivity growth rate, since it is this difference that matters in the calculation. The intersection of the demand and supply curves locates the equilibrium steady state capital intensity and interest rate (that is, the difference between the interest rate and productivity growth rate) in the economy, from which wages, income and consumption can be calculated. The bold lines in Figure 1 are a schematic representation of Tobin's figure. He recognized that aggregate demand for wealth by households could also be satisfied by transfer wealth

such as that generated by the Social Security system, leading to less capital and higher interest rates in equilibrium. Willis (1988) modified Tobin's diagram to show the effects of positive or negative transfer wealth on the equilibrium. This is indicated by the two dashed lines in Figure 1. Upward transfers like a pay-as-you-go public pension system generate positive transfer wealth which substitutes for capital in satisfying the demand for wealth, leading to less capital and higher interest rates in equilibrium (the dashed line to the left). Downward transfers like public education or planned bequests generate negative transfer wealth which augments the life cycle demand for wealth, and leads to more capital and lower interest rates in equilibrium (the dashed line on the right). Lee (1994 a and b) estimated the positive and negative transfer wealth arising from the public and private sectors in the US.

Tobin's analysis was comparative steady state. Lee, Mason and Miller (2001, 2003) simulated a similar model calibrated to the economic and demographic experience of Taiwan and the US for the demographic changes of the 20th century and projected into the 21st. They noted the doubling or tripling of the demand for wealth that resulted from population aging across Taiwan's demographic transition. They (Mason and Lee, 2006) called this the "second demographic dividend". (The "first demographic dividend" refers to the benefits arising from an improving support ratio in the middle of the demographic transition, which will be discussed later.) Some scenarios incorporated income sharing with elderly parents which generated transfer wealth and reduced the increase in asset holdings, similar to the simulated effect of Social Security in the US. They also noted that with serious population aging, savings rates fell at the same time that capital intensity increased, echoing the point made by Cutler et al (1990).

More recently, general equilibrium models with demographically realistic overlapping generations and perfect foresight have been used to simulate the effects of population aging and other demographic change with setups conceptually similar to the Tobin (1967) model (Borsch-Supan, Ludwig, et al. 2006; Sanchez-Romero, 2009). Romero incorporates public pensions and private old age support through sharing rules, as in Lee, Mason and Miller (2000, 2001), deriving richly detailed dynamic results. There is increased capital intensity across the demographic transition, but it is muted because declining interest rates (resulting from the assumption of a closed economy) lead to lower saving rates.

Another literature places greater emphasis on the effects of population age distribution across the demographic transition in the Third World, synthesizing models of changing support ratios with models of growth convergence (Higgins and Williamson, 1997; Bloom and Williamson, 1998; Bloom and Canning, 2001, 2002). This influential research suggests that population aging will indeed lead to declining aggregate saving rates, and points to the favorable age distribution trends in mid-transition as not only raising per capita income through mechanical composition effects, but also boosting savings rates -- the "first demographic dividend" mentioned above.. Kelley and Schmidt's (2005) synthetic empirical analysis in a convergence framework supports these findings.

Our Approach

In a closed economy, total net output, Y , is taken to be a function of the level of technology, A ; labor inputs in efficiency units, L ; capital per unit of labor, k ; and human capital per unit of labor, hk :¹

$$Y = L[Af(k, hk)]$$

Analysis of an open economy along the lines of our previous work would allow for additional income earned by residents from their foreign asset holdings or income flowing to foreigners who own domestic assets. Moreover, returns to capital and perhaps human capital would be determined in global markets rather than by the amounts of capital and human capital in the domestic economy. Here we focus on a closed economy.

Total consumption, C , depends on the proportion of net output saved, s ; on the support ratio (equivalent labor units, denoted L , per equivalent consumer, denoted N); and output per equivalent worker, Y/L . We are interested in consumption per equivalent consumer, $c=C/N$, which is given by:

$$c = (1-s)\left(\frac{L}{N}\right)Af(k, hk)$$

That is, consumption per equivalent adult consumer, c , equals the product of the proportion of output that is consumed, the support ratio, and output per worker. In analysis presented later in the paper the saving rate, s , can be inferred. We have explicitly modeled and simulated saving in a number of papers (Lee, Mason and Miller, 2000, 2001; Mason and Lee, 2007) but space limitations preclude treating it explicitly here. The level of technology, A , will be assumed to rise at a constant rate. Thus, we will focus our attention on three factors that determine c :

- The support ratio
- Capital per efficiency unit of labor, and
- Human capital per efficiency unit of labor.

These factors vary over the demographic transition with changes in age distribution, survival and fertility. They depend on age patterns of consumption both public and private. They depend on how labor income varies by age which in turn depends on labor supplied at each age, adjusted for efficiency, participation rates, and hours per participant. Capital per efficiency unit of labor depends, in addition, on the amount of wealth per unit of labor income that the population desires to hold, in order to achieve their planned levels of consumption and labor income in future years. This desired wealth can be held either as transfer wealth or as capital (in a closed economy), and we will consider the forms in which it is held in different countries and the level of demand for capital that results. Finally, we note that population aging has been driven mainly by low fertility, and we consider the association between the level of fertility and the level of investment in human capital per child across nations. National Transfer Accounts provide the empirical foundation for the analysis.

National Transfer Account Estimates of Labor Income and Consumption by Age

The National Transfer Accounts project (see <http://www.ntaccounts.org/>) provides cross-sectional estimates, in a manner consistent with National Income and Product Accounts, of all age-specific economic flows: public and private consumption, labor income, public and private intergenerational transfers, public and private asset income, and public and private saving. Research teams in twenty eight countries on five continents participate in the project and twenty three of these have produced at least the basic age profiles. We will draw on these, and on subsets of countries with more elaborate estimates, over the rest of this paper.

For our purposes, labor income by age is an average across all people of a given age, men and women, those in the labor force and those not, those employed and those self-employed, including unpaid family labor. It includes fringe benefits as well as direct payments (see Lee, Lee and Mason (2008) and Mason, Lee, et al. (2009)). To make our age profiles comparable across countries, we standardize each one by dividing by the average labor income for ages 30 to 49, ages chosen to avoid the effects of prolonged education and early retirement.

Similarly, consumption is averaged across all people of a given age, males and females combined. It includes private expenditures by households, which are imputed to individuals using equivalent adult consumer weights, after separately allocating private expenditures on health and education to individuals in the household directly from survey information, or indirectly using regression methods. Public in-kind transfers of education, health care, long term care, and pro-rated public and quasi public goods are also included. Then as with the labor profiles, the consumption profiles are standardized by dividing by the average labor income at ages 30-49 for that country.

These age profiles are currently available for 23 countries, but to simplify the exposition we plot, in Figure 2, the average labor income and consumption profiles averaged across four poor countries (Indonesia, Philippines, India and Kenya) and four rich countries (US, Japan, Sweden and Finland). First consider labor income. As we might expect, it is many times as high in childhood in poor countries as in rich ones, although labor income of children is very low in some poor countries.² It peaks earlier in poor countries, perhaps reflecting the more physical nature of work there, and the greater returns to experience for more educated workers. Finally, we see that labor income drops sharply in the rich countries after the late 50s, perhaps reflecting incentives built into pension programs (Gruber and Wise, 1998) and an increase in the demand for leisure induced by rising income (Costa 1998). Labor income remains substantially higher in the poor countries.

Turning to consumption, the high levels at young ages reflects human capital investment in the rich countries, and this bulge is absent in the poor countries. It is also striking that in the poor countries, consumption is quite flat across adult ages, from the early 20s until the end of life. In the rich countries, by contrast, consumption rises with age. In the US this rise partly reflects an increase in privately funded consumption with age, and partly reflects a strong increase in publicly funded health care and long term care with age.

These estimates of consumption can be subtracted from labor income at each age to calculate the cross-sectional “life cycle deficits” at young and old ages and the “life cycle surpluses” at middle ages. The remainder of our analysis considers the implications of these life cycle deficits and surpluses, and of the ways that consumption in the dependent ages is supported, through transfers or use of asset transactions (borrowing and lending, saving and dissaving, investing in assets and selling assets, receiving or paying interest, dividends or rents including imputed rent, and realizing profits and losses from a farm or business).

NTA includes detailed estimates of flows of public and private transfers by age, as shown in summary form for the US in 2003 in Figure 3. The portion of the cumulative plot above the horizontal axis (zero line) shows six kinds of inflows of transfers: public and private transfers for each of health care, education, and “other” (which includes long term care). The portion below this axis shows the outflows of transfers, that is taxes paid for public sector transfers, and private transfers made. Private inflows and outflows are estimated separately for interhousehold and intrahousehold transfers, but these are combined in the figure. The inflows and outflows can be summed to obtain the net transfer flows at each age.

These transfer data, augmented by data on asset income and saving (not shown in Figure 3), can be used to estimate the sources for funding the life cycle deficit of the elderly (that is, their consumption minus their labor income).

NTA Estimates of old age support

There are three important ways that the elderly can fund their lifecycle deficit (LCD65+), that is consumption above age 65 less labor income above 65. The elderly fund their consumption, to some extent, by continuing to work. But our interest here is in how the elderly fund their lifecycle deficit. The deficit can be met by relying on accumulated assets – a combination of asset income and dis-saving; by relying on public transfers such as pensions or health care to the extent that they receive more in benefits than they pay in taxes to support such programs; and by relying on private transfers, principally familial old age support as in some parts of the Third World. Figure 4 shows the proportion of LCD65+ that is funded by each of these three sources in each of twelve NTA countries. These proportions for each country must add to 1, so they can be represented on a triangle plot, as in the figure.³

The values in Figure 4 are for a recent year between 1998 and 2004. In three Asian economies, net family transfers to those 65 and older are positive ranging from about 20 percent of the lifecycle deficit in South Korea to about 40 percent in Taiwan. In all other cases, net family transfers are essentially zero (Japan and Finland) or negative (Philippines, Mexico, US, Uruguay, Costa Rica, Germany, and Austria). In some cases net family transfers are positive at ages older than 75 or 80 (Mexico, Japan, Costa Rica) but not in the United States or any of the European countries shown in Figure 4. (Note that the magnitude and direction of familial flows in developing countries is not closely tied to the extent to which the elderly live with their descendants.)

The importance of public transfers for the elderly varies substantially around the world. Net public transfers are essentially zero in the Philippines and Thailand. Among the advanced industrialized countries the elderly in the US fund under 40% of their deficit from net public transfers, while in Germany, Finland, and Austria net public transfers fund more than two-thirds of the lifecycle deficit.

In the Philippines, Mexico, Thailand, and the United States, a surprisingly disparate group, two-thirds or more of the lifecycle deficit of the elderly is funded by relying on assets. In Taiwan, Germany, Finland, and Austria the elderly fund less than one-third of their lifecycle deficit relying on assets.

Support Ratios and Demographic Change

The support ratio is the ratio of effective labor to effective consumers. Effective labor in year t is defined as the sum of the population age distribution for that year and a standard labor income age profile. We use NTA data to generate this standard, as described below. Effective consumers are defined similarly, drawing on NTA consumption age profiles. We don't know how age profiles of industrial nations will change in the future. Quite possibly, labor supply at older ages will rise. Quite possibly, the costs of health care at older ages will continue to rise, but costs of long term care may decline as the health of the elderly improves. Here, we simply assume that the average age profile for the richer countries shown earlier will continue to hold in the future, and use it for the US, Japan and Spain. We also use it for these countries for the historical past back to 1950. For the Third World countries, we take their individual current age profiles for 1950 to 2010. For the future, we assume they are trending linearly toward the rich country average profile, with different pre-specified dates of arrival. For the population age distributions we have relied on United Nations data.⁴

Consider the demographic changes over the course of a classic demographic transition. Initially, mortality begins to fall while fertility remains high, resulting in rising child dependency ratios as more births survive. Rising child dependency translates into falling support ratios. Eventually, typically after a number of decades, fertility begins to decline and child dependency falls, causing the support ratio to begin to rise. This phase of rising support ratios continues for three to five decades, producing the first demographic dividend as support ratios rise well above their pre-transition levels. Eventually, however, fertility decline leads to slower labor force growth and mortality decline leads to more rapid growth of the elderly population. Population aging reduces support ratios which may end up close to their pre-transition levels, or perhaps even below them if fertility is far below replacement. In the industrial nations, this classic transition was disturbed by baby booms in the 1950s and 60s followed by baby busts, generating more complex patterns.

The results are shown in Figure 5, which plots the support ratios for three industrial nations (the US, Japan (JP), and Spain (ES)) and for three Third World nations (China (CN), India (IN) and Kenya (KE)). In this phase support ratios decline, as we see for Kenya, China and India. Fertility decline came latest and has proceeded most slowly in

Kenya, which still has a TFR near 5. Fertility decline started in China and India around the same time, but decline was much more rapid in China, which shows in its steeper and higher increase in the support ratio, followed by a steeper and earlier onset of population aging and falling support ratios. In India, the first dividend phase will apparently continue for another few decades, while in China it has already ended.

Among the industrial countries, Japan's fertility decline came later than that of the US or Spain, but the US had a major baby boom and subsequent fertility near replacement, while Spain's fertility decline stalled for decades before resuming. As a result, Japan's support ratio was the first to fall due to population aging. The decline in the US starts around 2007, and in Spain about five years later. The decline in the US is more modest due to its higher fertility, while in Japan and Spain the support ratio plunges.

How important are these swings in the support ratio? Given the uncertainties surrounding long term forecasts, let's focus on the projected changes between 2010 and 2050. For Kenya, the rising support ratio would by itself, other things equal, raise age specific consumption by .6% per year. For India, this is .2% per year, and in China, aging will drop consumption by .4% per year. Given the growth performance of China and India in recent decades, these positive and negative effects are modest, but for Kenya the gain might matter more. All three industrial nations will experience declining support ratios. For the US, this is .2% per year; in Japan .7%, and in Spain .8%, between 2010 and 2050. The US decline (similar to that reported in Cutler et al, 1990) is small and slow. Because US fertility is relatively high, population aging will be less severe. However, for Spain and Japan, the anticipated decline in the support ratio is three or four times as large and quite important.

These calculations tell us what would happen if current consumption and labor supply remained similar in the future. However, they will doubtless change in the future for many possible reasons: labor supply at older ages may increase, health care costs of the elderly may continue to rise faster than productivity growth, the involvement of families in care of their elderly may rise or decline, and so on. Most governments will attempt to modify publicly funded consumption at older ages and to raise the age at retirement.

Increases in the support ratio during the demographic transition may be used to fund increased consumption, or alternatively to fund investment in physical or human capital, thereby making a lasting contribution to economic growth. Tightening of the budget constraint due to population aging may be systematically linked with intensification of human or physical capital. We will now explore these connections.

The demand for wealth and producer's demand for capital

Individuals choose to hold wealth – claims on future output in excess of their future labor income – for many reasons. They may want to save for their retirement, or more generally to smooth their household consumption over the normal ups and downs of labor income over the life cycle. This is what we call the life cycle demand for wealth. But there are other motives as well. They may want to accumulate a buffer fund in case of future emergencies. They may want to leave a bequest for their children. Or they may just

enjoy the feeling of power it conveys. Whatever the reason, the demand for wealth and actual holdings of wealth vary strongly by age, and changes in the population age distribution exert a powerful effect on the aggregate demand for wealth, as we shall see.

Wealth can be held in two basic forms: capital or transfer wealth. For the moment, let us assume that all wealth is held as capital, and restrict our attention to life cycle wealth, needed solely for the purpose of smoothing consumption and providing for retirement. In this case, the amount of wealth that the average adult in the population wants to hold is equivalent to the amount of capital they want to hold, at some given interest rate and rate of productivity growth. Figure 6 plots this amount as $W(r)$, assuming a productivity growth rate of 2%. The curves shown here have been calculated based on the cross-sectional consumption and labor income age schedules for three countries differing in various ways including population age distribution: the US, Japan, and the Philippines. The calculated life cycle wealth assumes that adults smooth their individual consumption, while planning to provide for the consumption of their children at the level indicated by the consumption age schedule. Each adult expects productivity growth at rate 2% to continue into the future, lifting their age-earnings schedule, and likewise lifting the age-consumption profile of themselves and their dependents. They set the level of this consumption profile such that the life time present value of their expected labor income will equal the present value of the consumption for which they pay, including their dependents.

Figure 6 plots this curve for values of r from 0% to +10% per year. We see that $W(r)$ is positive in this range for the United States and Japan, but mostly negative for the Philippines. We also see that in the Philippines with a young population, the curve is located to the left; in the US with an intermediate age population, it is in the middle; and in Japan, with an old population, it is located farther to the right.

This curve describes the funds for investment that the population would offer at each r , purely for purposes of life cycle smoothing, under the unrealistic assumption of no public or private transfers. Producers also want to invest in capital to increase production, with the amount depending on r . This is a rectangular hyperbola if the production technology is Cobb-Douglas with constant returns to scale (see Appendix). As drawn here, this curve assumes that there is an equity premium of 1% (this value is chosen solely to produce equilibria for the US and Japan, for purposes of illustration).⁵ The intersection of the two curves locates the equilibrium interest rate and ratio of capital to labor income, on the assumption that capital markets are closed.⁶

For the Philippines we see that there is no intersection, so producers' demand for capital would have to be satisfied by foreign investors, counter to the closure assumption made here. Both Japan and the US have two equilibria, one with little capital and a high interest rate which is unstable, and another with much capital and lower r , which is stable. We will focus on the stable equilibria to the right. The US equilibrium is at about $r = .02$ and $K^*/YI = 6$. The Japan equilibrium is at about $r = .01$ and $K^*/YI = 10$. More realistically, these are open economies and we could also simply show a horizontal producer's demand schedule at a fixed interest rate.

The equilibria in the figure should be regarded as more conceptual than empirical, illustrating that population aging leads to increased capital intensity.

Figure 7 shows the ratio of life cycle wealth W , calculated in this way, to average labor income for 23 countries, for a discount rate of 3 percent and expected productivity growth rate of .02. This ratio is plotted against the proportion of the population 65+ as a measure of population aging. We see that the association between the demand for life cycle wealth and population aging is strong ($R^2=.85$ in the descriptive regression). Further analysis indicates that between two thirds and three quarters of this association is a pure effect of population aging, while the remainder is due to the change in the shapes of the age profiles of consumption and labor income that we noted earlier. The figure shows an increase from around 0 in the standardized demand for wealth in the youngest populations to around 7 in the oldest population. This increase, if translated into increased holdings of capital, would imply an increase in labor productivity by a factor of two or three.⁷

However, as discussed earlier, wealth can be held in other forms besides capital, so this calculation is a great oversimplification.

Transfer Wealth and the Demand for Capital

Earlier, Figures 3 and 4 showed various aspects of our NTA estimates of public and private flows of transfers. We can re-express these flow data as a form of wealth, by assuming that individuals expect these cross-sectional patterns to hold in the future, albeit modified by expected productivity growth.⁸

Transfer wealth at age x , denoted $T(x)$, is the present value of survival weighted transfers expected to be received in the future, minus those expected to be paid. Private transfer wealth could be the expectation that adult children will provide support to retired parents. Public transfer wealth could be the expectation of Social Security benefits net of contributions. Bequest wealth is the difference between expected receipt of bequests in the future and expected bequests left to others. This will be negative for the population as a whole, because people on average receive bequests at a younger age than they make them. Thus the expectation of leaving bequests leads to a greater demand for capital. In our estimates, bequest wealth is a catch-all category that expresses the effects of all motivations for holding wealth beyond life cycle saving as modified by public and private transfers. If people accumulate wealth just for the thrill of it, then when they die they will leave it as a bequest, intended or not, which we will measure.⁹ The portion they consume already enters into the life cycle wealth calculation by way of the average $c(x)$ age profile. Likewise if they do not annuitize their wealth, and so save extra for retirement in case they live longer than average, this will lead to larger unintended bequests, which we can measure.

Figure 8 (based on Tobin 1967 and Willis 1988 and US NTA estimates for 2003) shows for the US how private transfer wealth, public transfer wealth, and bequest wealth are

combined with life cycle wealth to determine the amount that the population wishes to invest in capital at each interest rate. The left panel shows each form of wealth separately in relation to r . In the right panel, the demand for capital by producers is calculated from a constant returns to scale Cobb-Douglas production function with a capital elasticity of .25, and with an assumed equity premium of .01 above the discount rate r . The demand for life cycle wealth is $W(r)$. The equilibrium based on the pure lifecycle demand for wealth with no transfers or other motivation to hold wealth is at the intersection marked A, at roughly $r=2\%$ and $K/Y=6$. The second curve shows the full demand for capital taking all transfers into account, given by the sum of the four curves plotted in the first panel. The intersection of this curve with the demand for K curve gives us the equilibrium value of $r=.5\%$ and of $K/Y=13$. These are not realistic values, but the different equilibria illustrate the important role of transfers in determining the equilibrium interest rate and capital stock.

One clear implication of Figure 8 is how misleading it could be to focus on the life cycle saving motive for asset accumulation without attending to public and private transfers as well. We can assess the relative size of the various motives if we specify a value for r (assuming an open economy, perhaps) and examine the left panel. For $r=3\%$ the left panel indicates that the life cycle demand for wealth (relative to per capita labor income) is +4.8, private transfer wealth other than bequests is -3.0, public transfer wealth is +2.3, and bequest wealth is about -2.5. Recall Kotlikoff and Summers (1981) who found that a greater proportion of the US capital stock could be explained by the desire to make intergenerational transfers than by the life cycle saving motive. Combining the two components of private transfer wealth we come to -5.5, which contributes more to the demand for capital than life cycle wealth at 4.8. However, once we also bring public transfer wealth into the picture it negates some of the private transfer wealth, leaving a net value of transfer wealth of -3.2 versus 4.8 for life cycle wealth, and generating a total demand for wealth of 8.0 instead of 4.8. This is at least partially in line with a Ricardian Equivalence interpretation of the relation of public to private transfers (Barro, 1974; Feldstein, 1974).

Population Aging, Capital Accumulation, and Consumption Levels

We have seen that a population has a certain demand for life cycle wealth, which is the per capita level of wealth needed to smooth consumption over the life cycle including the consumption of children, and provide for consumption during retirement. Population aging drives massive increases in this demand for wealth, because with a greater proportion of elderly, more wealth must be held to provide for their old age consumption. However, this does not necessarily translate into an increased demand for capital, since life cycle wealth can be held as public or private transfer wealth as well as in the form of assets. Furthermore, there are other motives for holding wealth besides consumption smoothing, and these inevitably lead to bequests, whether intended or unintended.

Population aging across the demographic transition, including those later stages of aging that many populations will experience over the coming decades, shifts the demand for wealth and the demand for capital to the right, leading to increased capital per worker, and therefore higher productivity. This brings us back to the expression for productivity

per worker: $Af(k, hk)$. The increased demand for life cycle wealth, other things equal, entails an increase in k and therefore increased productivity. The extent to which this occurs depends on the extent to which transfers are used to fund consumption by the elderly.

Our discussion of these points has been based on comparative steady states, and various other simplifying assumptions. However, we have also done dynamic simulations (Mason and Lee, 2007) of consumption, savings, and assets over the demographic transition, based on the assumption that the share of transfers in funding the old age life cycle deficit remains unchanged, and that the cross-sectional consumption profiles also keep the same shape although their levels change over time. In these simulations, we find a substantial increase in the ratio of assets to labor income, along the lines suggested earlier. These simulations adhere closely to the measures and concepts introduced above. In an earlier study, rather than using the NTA consumption age profiles, we assumed that adults are strict life cycle savers, in a dynamic version of Tobin's 1967 study (Lee, Mason and Miller, 2000, 2001). That study showed a doubling or tripling of the demand for wealth due to the population aging that accompanies the demographic transition in Taiwan. We also found that increased reliance on familial transfers or public sector transfers substantially diminished this increase in the demand for wealth.

Finally, Romero (2009) develops a general equilibrium OLG model with full demographic structure, taking the demographic transition as exogenous, and assuming perfect foresight. He incorporates familial support of the elderly and public sector pensions to specifiable degrees. He finds capital intensification over the course of the demographic transition, but to a lesser degree when there are familial or public transfers to the elderly. He can also simulate pre and post transitional demographic steady states, and generate from these diagrams much like the Tobin-Willis diagrams (Figures 1, 6 and 8) showing the forms in which wealth is held, the demand for capital, and the equilibrium capital stock and interest rate. In his closed economy model he finds smaller effects on the demand for capital than we have in an open economy model of Taiwan, because increased capital intensity drives down the rate of return on assets and dampens saving rates.

Population Aging and Human Capital

Human capital shares many of the features of capital that are discussed above. Human capital is productive and, hence, investment in human capital is a way that producers can increase their output. Human capital has a role in dealing with the economic lifecycle. Working age adults can invest in their children and when old rely on their children for old-age support (Becker and Tomes 1976). Governments can act in a similar fashion by providing public support for human capital spending and taxing the enhanced earnings of the beneficiaries to support the elderly (Becker and Murphy 1988). Investment in human capital may also be motivated by concerns completely unrelated to the lifecycle. For example, parents may invest in the education of their children for altruistic reasons, leading to the creation of human capital that is transmitted from generation to generation. (Note that the transmission process is somewhat more complex with human capital because it is embodied. Individuals can't bequeath their human capital, but they can

invest the returns from human capital into the human capital of their children and thereby accomplish much the same thing.). Parents may derive utility directly from the accomplishments of their children, and invest in it.

Since the pace and extent of population aging is largely determined by the level of fertility, we are particularly interested in the relation between fertility levels and investment in human capital (see Lee and Mason, 2009). Under the quantity-quality theory (Becker and Lewis, 1973; Willis, 1973), parents allocate their resources among own consumption, numbers of children, and quality of children, where quality is sometimes defined as quantity invested in the average child. In simple versions, parents first choose the share of income to spend on own consumption, and then decide how to allocate the remainder between quantity and quality of children with the product of numbers of children and average investment per child entering the budget constraint. Taking the budget share devoted to children as fixed, the elasticity of quality with respect to quantity would be -1. Nothing here tells us whether exogenous fertility variation is influencing human capital investment, or the reverse, or perhaps another factor like income growth is influencing both.

Across countries, we can standardize expenditures on children by dividing by the average labor income across ages 30-49 as was done elsewhere in this paper. If the share of labor income that is spent on children is similar across countries, then we would also find a quantity-quality elasticity close to -1 cross nationally. We provide these comments as a benchmark or frame of reference for interpreting the results.

Here we will focus on human capital investments rather than general expenditures on children, most of which are for ordinary consumption (basic lodging, food, clothing). We measure human capital investment in a child at age x as the sum of private and public spending per child on health and on education. We then sum this measure over ages 0 to 17 for health and 0 to 26 for education to find total human capital investment per child, HK, in a synthetic cohort sense, based on our cross-sectional data. We divide this amount by average labor income for ages 30-49.

The left panel of Figure 9 plots the log of HK against the log of the Total Fertility Rate (TFR) for the five years before and including the survey year for the NTA estimates. Clearly the relationship is negative. The elasticity of HK with respect to TFR in a descriptive regression is -.9, fairly close to the -1 benchmark.

For a few countries, Japan, Taiwan and the US, we can do a similar analysis over time, as shown in the right hand panel.¹⁰ Here we find elasticities of -.7 for the US, -1.5 for Japan, and -1.4 for Taiwan (Ogawa, Mason et al. 2009).

While direction of causality is unclear, these data indicate a negative relationship between fertility and human capital investment per child relative to labor income. Because low fertility is the main demographic source of population aging, we would expect to find a similar relationship between population aging and HK per worker or per

capita. Descriptive regressions confirm these relationships both cross-sectionally and over time within countries.

In Lee and Mason (2009) we develop a simple OLG model that incorporates this estimated relationship between HK investment and fertility, and additionally includes an effect of HK of workers on their productivity. There is no physical capital. In this model lower fertility leads to fewer workers, an older population, and a higher old age dependency ratio. But it also leads to more investment of HK in children, so that each of the fewer workers is more productive, tending to offset the effects of the rising old age dependency ratio. We simulate the economic outcome over a stylized demographic transition. Initially the support ratio rises as fertility falls. However, the higher support ratio now leads to higher investment in HK, so the first demographic dividend is partially invested. This leads to subsequent increases in worker productivity, and even with population aging consumption is higher in the long term.

Conclusion

Low fertility and rising longevity will cause the proportions of elderly to rise strongly in all countries during the 21st century. Even the oldest national populations of today will experience a doubling or more in their old age dependency ratios in the next decades. There has been a tendency to focus on the difficulties this aging will cause for government programs for the elderly, and particularly public pensions. But public pensions cost only a small fraction of national output, and focusing on them can easily give a mistaken sense of the consequences of rising old age dependency. The changing support ratio, which reflects much more than pensions, and which is more realistic than the dependency ratio, gives more useful information. We have drawn here on NTA estimates of consumption and labor income across the age distribution to construct support ratios for a selection of countries. Viewed through support ratios, we see that for a number of countries the changes expected from 2010 through 2050 are relatively unimportant in some countries, like the US (where fertility is near replacement) and India (where population aging toward 2050 will neutralize some of the earlier gains). For others, like Japan and Spain, support ratios will decline faster and farther, reducing growth rates of age-adjusted consumption by nearly 1% per year. For Kenya, the gains from a rising support ratio will be important at about .6% per year.

Population aging leads to falling support ratios. Other things equal, that would mean falling consumption. However, we have suggested here that population aging and the low fertility and mortality that cause it can themselves generate increased income by raising the accumulation of physical capital and human capital, or through increased investments in foreign assets.

First consider physical capital. The demand for wealth, or which capital is one form, arises from diverse motivations, but the desire to provide for consumption during retirement is among the most important. We have shown that this life cycle demand for wealth increases greatly as populations grow older. If most wealth were held in the form of capital, then population aging would drive a great increase in holdings of capital relative to labor income. However, public and private transfers, including bequests, and

the corresponding forms of transfer wealth, substitute for capital in satisfying the aggregate demand for wealth, or in some cases augment it. Using NTA data we find wide cross-national variation in the roles of assets and public and private transfers in funding the consumption of the elderly. These must be considered in conjunction with the demand for wealth to understand the forces driving domestic capital accumulation and international capital flows.

For quite different reasons, the low fertility that drives population aging also is strongly associated with rising rates of investment in human capital per child, which in turn raises labor productivity.

The demographically induced increases in capital and human capital per worker could easily overwhelm and reverse the negative effects of population aging working through the support ratio. Whether or not they do depends in part on policy. Heavy reliance on public or private transfers to provide for consumption in old age reduces the beneficial effects of population aging. Ill-functioning financial markets and a financially illiterate work force would reduce them as well. Ineffective educational institutions could also fail to translate a demand for education into increased labor productivity in later years.

Many benefits of the demographic changes that produced population aging have already been with us for decades, and we largely take them for granted. Rising payroll taxes and pension reform are painfully present and clearly linked to population aging. Deeper capital stock and a better educated work force do not make these visible costs of population aging go away. However, if we take a few steps back then we can see that population aging brings economic benefits that may be at least as important as its costs.

¹ We assume constant returns to scale. The function f gives output per worker for a unit level of technology. Labor in efficiency units takes into account pure effects of age on the productivity of labor, assuming the same level of human capital at all ages. The effect of human capital on labor productivity is explicitly taken into account in the function f , on the assumption that only the average amount of human capital matters, and not the age of the workers in which it is embedded, or whether it is concentrated in a few or diffused across many.

² In preliminary NTA estimates the labor income of children and young adults is very low in Senegal, South Africa and Nigeria.

³ The gridlines and the axes (triangle sides) show movements between two sources holding the share of the third constant. Along the axes, the share of the third funding source, is zero. Movements along the horizontal axis, for example, would represent countries relying on family and public transfers to varying degrees and assets not at all.

⁴ We use the 2008 revision of the United Nations data and projections to 2050. We splice these to United Nations long term projections for 2050 to 2100.

⁵ More plausibly, the equity premium is 3 or 4%. Alternatively, we could change the elasticity of substitution of labor and capital in the production function to achieve an intersection of schedules at an equilibrium point. Obviously, this exercise should be viewed as illustrative.

⁶ For more details about the construction and interpretation of this diagram, see Tobin (1967).

⁷ The corresponding ratios to labor income, wL , would be twice as great, ranging from 0 to 14. Of this, about 2/3 reflects an increase due to population aging alone, or 9.7. Suppose that this increase in life cycle wealth lead to a corresponding increase in the capital labor ratio, K/wL . Assuming a capital coefficient of 1/3 in a Cobb-Douglas production function (see Appendix), the productivity of labor would increase in proportion to the square root of this, or by a factor of 3; for a capital coefficient of 1/4, the factor would be about 2. These figures suggest that going from 5% elderly to 20% elderly would raise output per unit of labor by a factor of 2 to 3, due to increased capital intensity.

⁸ In this calculation, we have not allowed for the fact that future population aging will mean that these individual expectations cannot be realized, for example due to fiscal unsustainability under population aging. We could impose a sustainability constraint on their expectations, but how individuals form their expectations in this regard is an open question.

⁹ If they consume it, that consumption will already be reflected in our consumption age profile, and only the excess of average wealth beyond average consumption will show up as a bequest in our calculations.

¹⁰ For the US, we have one early observation at the peak of the baby boom in 1960, followed by many other observations for the period when fertility was much lower and did not vary much over time. Our estimated elasticity for the US is based on the contrast of 1960 to later years, not on the variations among those later years.

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Equation Chapter 1 Section 1 Appendix. Basic results for the Cobb-Douglas case

Consider a constant returns to scale Cobb-Douglas production function:

$$(1.1) \quad Y = AK^\alpha L^{1-\alpha}$$

Without loss of generality set A to 1. Several well-known properties of the Cobb-Douglas production function are useful. Output per worker (y) is given by $y = k^\alpha$ (where $k=K/L$). The wage rate is the marginal product of capital, $w = (1 - \alpha)y = (1 - \alpha)k^\alpha$, and total labor income is $wL = (1 - \alpha)Y$. The rate of return to capital is the marginal product of capital or $r = \alpha Y/K$ and the total income accruing to capital is $rK = \alpha Y$.

The capital output ratio and output per worker

We want to find output per worker, y , as a function of the ratio of capital to total labor income or $\kappa = K / wL = k / w$. Substituting for w from above yields

$\kappa = k / w = k / ((1 - \alpha)k^\alpha) = k^{1-\alpha} / (1 - \alpha)$. Noting that $y = k^\alpha$ and rearranging terms yields:

$$(1.2) \quad y = ((1 - \alpha)\kappa)^{\frac{\alpha}{1-\alpha}}$$

Output per worker varies as the capital-output ratio, κ , to the factor $\alpha/(1-\alpha)$. For $\alpha=1/3$, y varies as the square root κ and for $\alpha=1/4$, y varies as the cube root of κ .

Producers' demand for capital

The producers' demand for capital (relative to total labor income) is found by dividing total capital income by total labor income and rearranging terms:

$$(1.3) \quad \frac{K}{wL} = \frac{\alpha}{(1-\alpha)} \frac{1}{r}$$

The producer's demand for capital relative to labor income is therefore either 1/2 or 1/3 times $1/r$, when α is either 1/3 or 1/4.

Several adjustments are required to complete the analysis. First, the rate of return to capital exceeds the lower risk rate of return r used by lifecycle planners by the equity premium. Thus, the producers' demand for capital is shifted downward by the assumed value of the equity premium when plotted against r .

Figure 1. Producer's demand for capital and households' supply of funds for investment in closed economy, after Tobin (1967), with three illustrative demand schedules for wealth

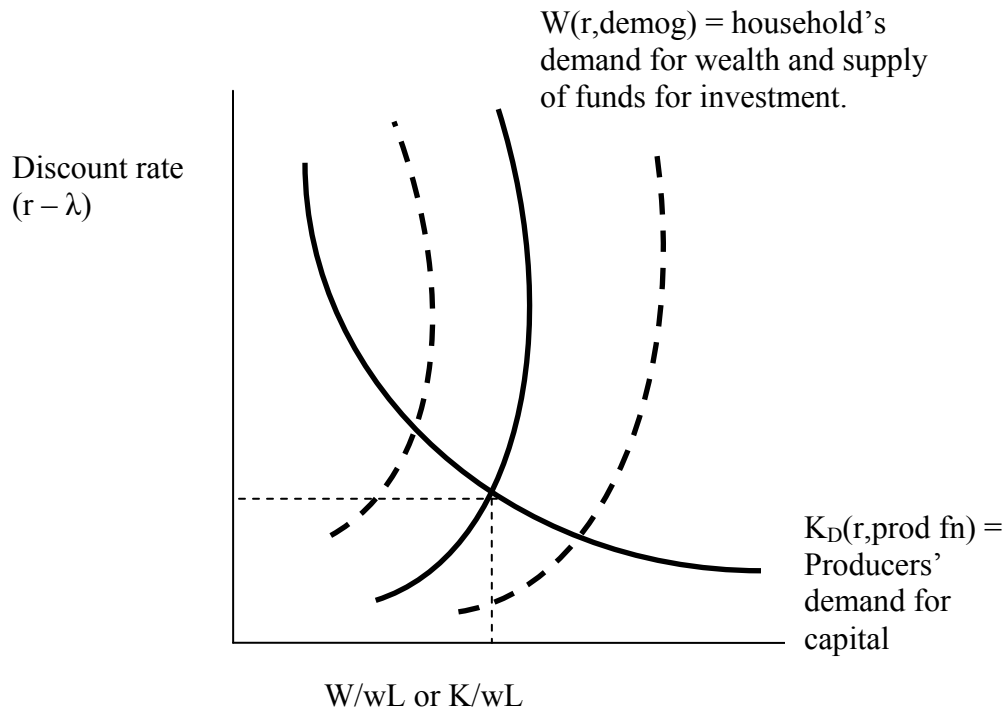
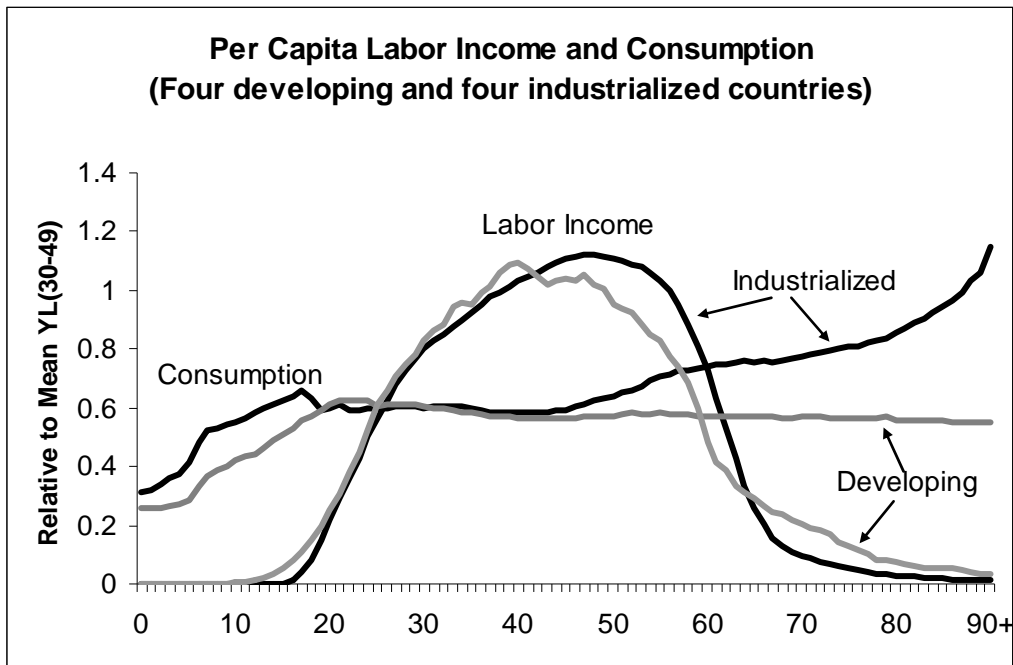
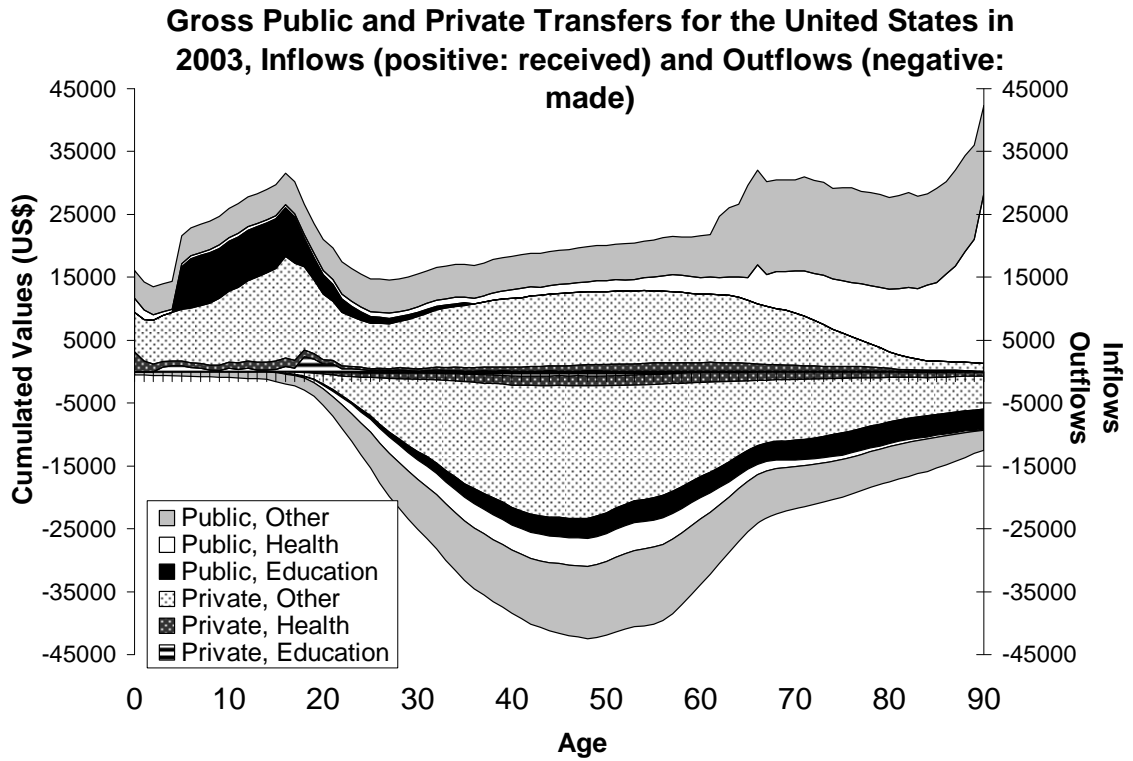


Figure 2



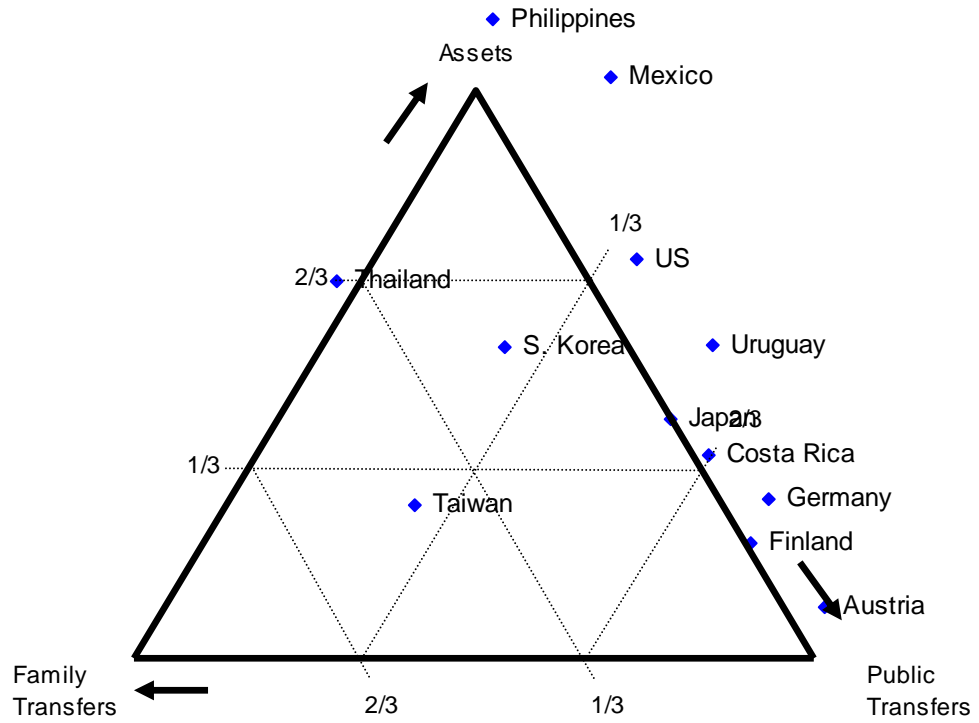
Notes: Four industrialized countries: Japan, Germany, Sweden, US;
Four developing countries: India, Indonesia, Kenya, and the Philippines.
Source: www.ntaccounts.org
From Av_Profiles_Sept2008_v3, tab Fig1 with 4 (2).

Figure 3.



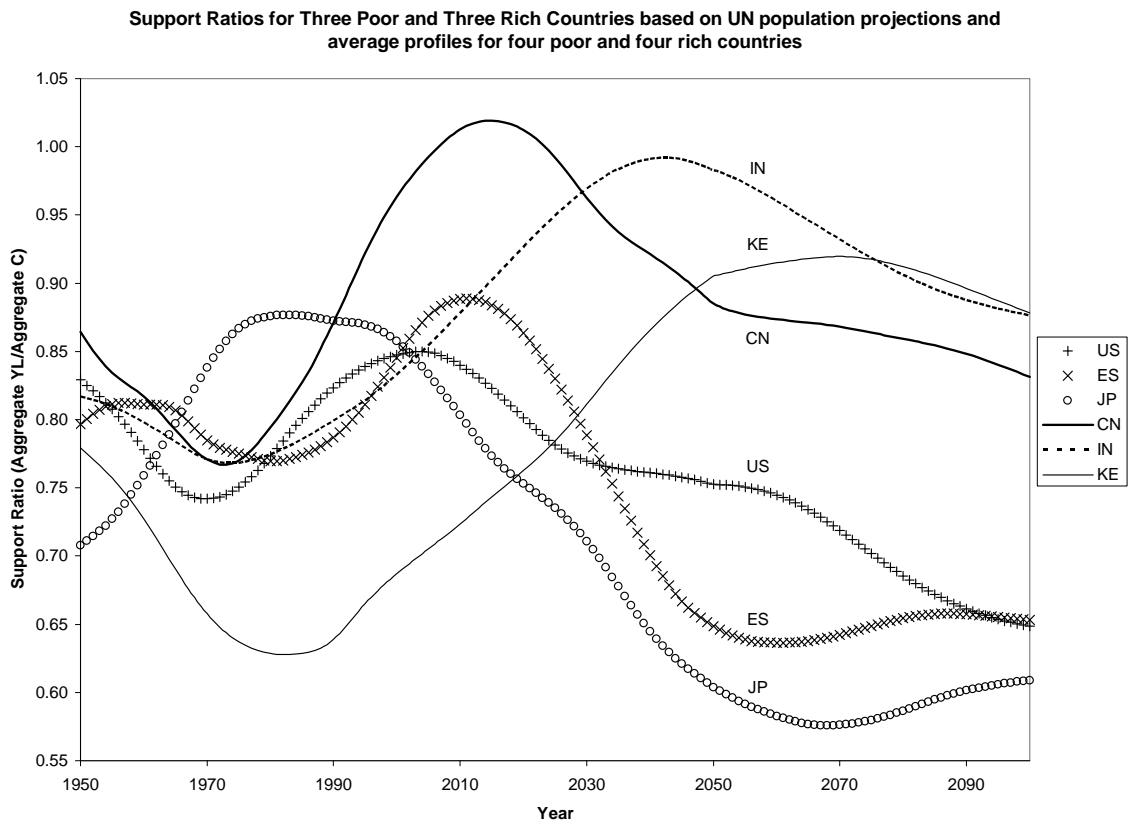
From Modified_Figure3_RDL

Figure 4. Share of Lifecycle Deficit of those 65 and Older Funded by Asset-based Flows, Public Transfers, and Familial Transfers, NTA Countries, 1998-2004.



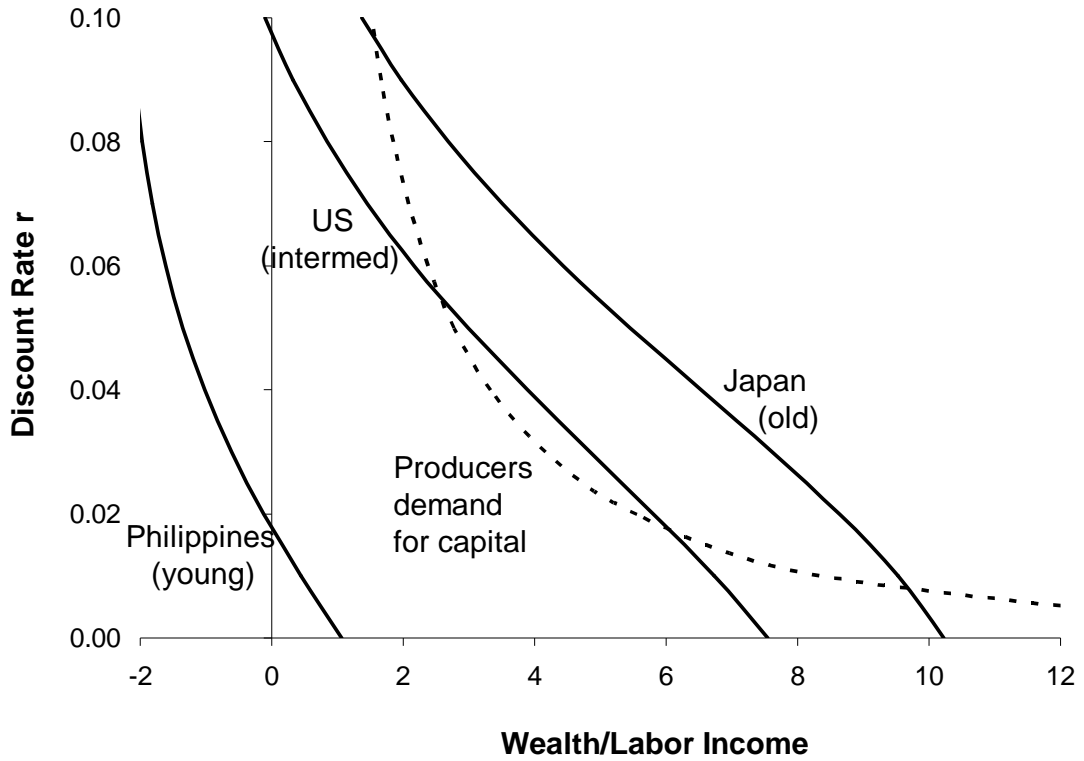
From Andy on Aug 12, 2009; no spreadsheet or source file.

Figure 5.



From SuppRatiosWithKenya workbook, Aug 12, 2009.

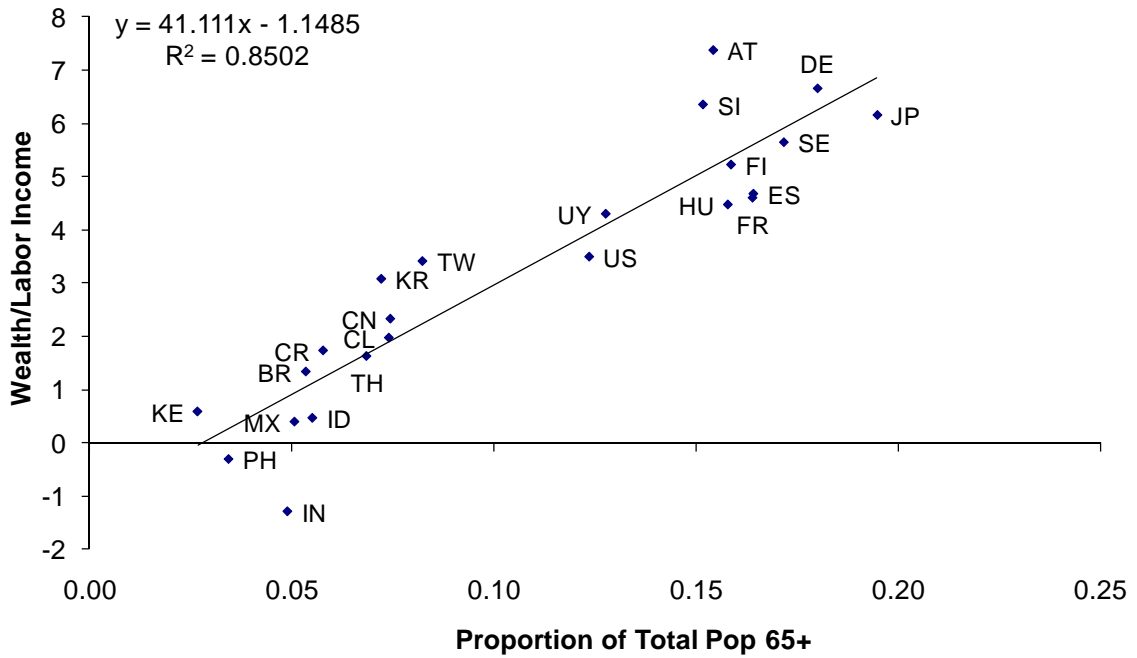
Figure 6. Population aging raises the aggregate demand for lifecycle wealth, $W(r)$. Demand for wealth schedules, relative to labor income, for three countries, young, intermediate and old, together with producers' demand for capital schedule (relative to labor income) assuming an equity premium of .01.



From Figs_6_and_8_Oct13.RDL

Figure 7

**Ratio of Life Cycle Wealth to Labor Income vs Proportion 65+,
Based on Adult Calculations**



Note: This calculated value of wealth is based solely on adults, and child consumption is folded into adult consumption. This eliminates children's demand for credit (negative wealth). The results are qualitatively similar when wealth is calculated for all individuals including children.

From Adult_Wealth_Calc_June2009_NTA

Figure 8A. Forms of Wealth in the US: Lifecycle (W), Family transfers (Tf), Public transfers (Tg) and Bequests (Bq) (individuals discount at 3% and expect 2% productivity growth)

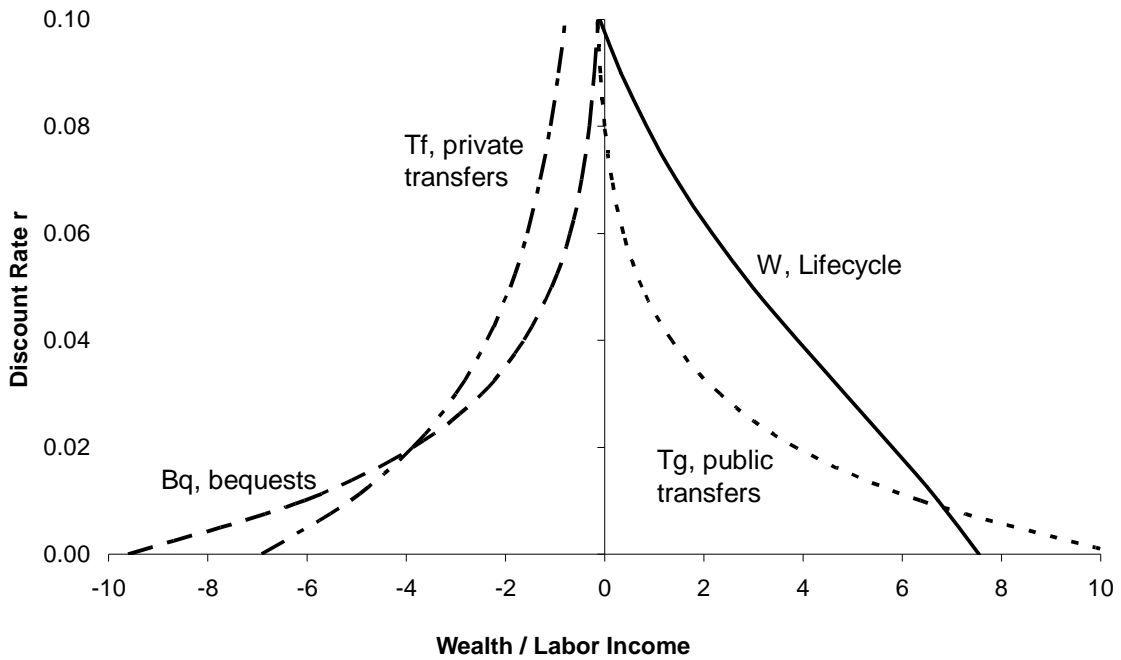
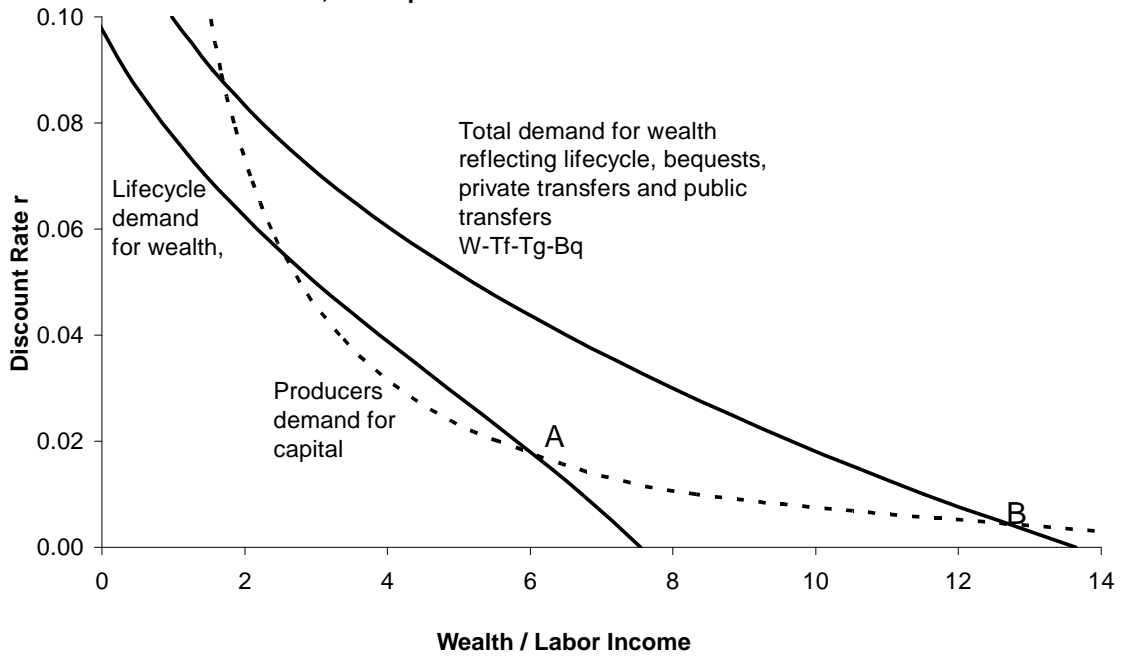


Figure 8B. Equilibrium in the Capital Market for the US in 2003, assuming a closed economy with 1% Equity Premium: A is equilibrium for pure life cycle wealth; B is equilibrium for total demand for wealth

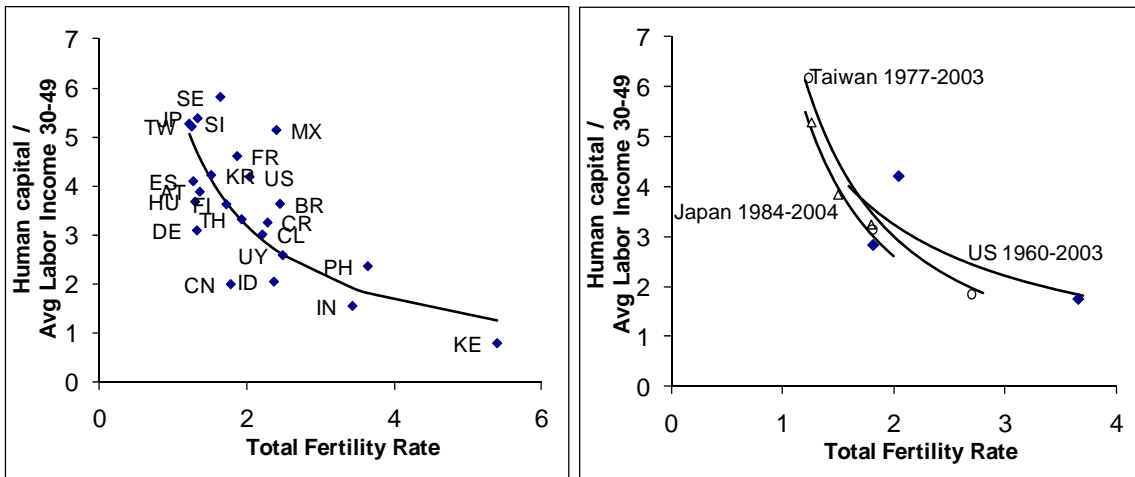


Note: A denotes the equilibrium when there are no alternatives to capital to satisfy the life cycle demand for wealth, and no bequests. B denotes the equilibrium ratio of capital to labor income for the full demand for wealth including the lifecycle demand

as well as public and private transfer wealth and bequest wealth. There are also equilibria with higher K and lower r , but these are unstable.

From Figs_6_and_8_Oct13.RDL

Figure 9. Relation of fertility to human capital spending per child in cross-section and time series



Note: Human capital spending is the sum of average age specific public and private spending per child for health and education, summed over ages 0-17 for health, and 0-26 for education. The total is divided by the average labor income for each country and period for ages 30-49. The total fertility rate is for the 5 year period closest to the year of the human capital estimate.