Endogenous Longevity and Economic Growth

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Abstract

In a two period overlapping generations model of endogenous longevity and economic growth, individuals choose to invest in health and education. The investments are costly in terms of foregone first period consumption and the benefit is in the second period where health has the effect of increasing the probability of survival, and education investment will bring higher income. These investments are risky as survival through period two, when the payoffs can be had, is not certain. Individuals with varying degrees of risk aversion will choose the ordering in which they invest in health and education. It is only when investment in education is achieved that an economy will experience endogenous growth.

Keywords: *Endogenous Longevity, Endogenous Growth, Health, Risk*

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1 Introduction

The relationship between life expectancy and economic growth is a newly contested field. Economic growth models that treat longevity as an exogenous parameter show that an increase in life expectancy will increase the time horizon over which returns to education can be realised, thereby encouraging investment in human capital and driving endogenous growth. Existing models of endogenous life expectancy, where the survival probability is a function of a choice variable within the model, also show this same positive relationship between life expectancy and economic growth. Moreover, models of endogenous longevity can explain a persistent low level of per capita income. By making not only education levels a choice of the individual, but also allowing the individual to invest in their health and influence their own life expectancy adds a new dimension to the analysis as an individual has control over the level of investment in the risky asset and also the probability of achieving that asset.

In this paper, the relationship between health and education, and thus the nexus with economic growth, is explored in detail, a task that has not been done to date. Investment in schooling has the effect of increasing income in the second period of life, but an individual will only live into this period probabilistically. Thus lifetime income is state contingent - a person lives through the second period or dies prematurely at the end of the first period - and if the individual lives through the second period then they are able to enjoy the higher income from skilled wages, but if they die they will have foregone current consumption to invest in schooling but are not alive to enjoy the benefit. Investment in education has the effect of increasing the return on the risky asset, but with health also a choice variable, the individual can also determine the probability of achieving the risky asset (second period consumption). An individual will compare this 'gamble' with a position of certainty, where investment in health and education remains at zero, when making decisions over whether to invest in the two choice variables.

Investment in schooling is sufficient for long run growth to prevail even when the productivity of human capital is low, however it is costly in terms of forgone first period income and the individual will not invest in schooling unless the expected utility of this investment is higher than the expected utility of zero investment in schooling. That the individual has command over the probability of living through the second period by investing in health, elucidates the effects of variable life expectancy on investment in schooling and thus economic growth. Understanding that with uncertain lifetimes individuals make decisions with state contingent outcomes, in which they not only control how much to invest in the risky asset but also make a choice over the probability of gaining the risky asset, sheds new light on our analysis of the incentives to invest in health and education and their flow on effects to economic growth.

In a survey of the literature pertaining to demographic change and economic growth, Ehrlich and Lui (1997) emphasise the next logical advancement is to develop models of endogenous longevity. Although Grossman (1972), Ehrlich and Chuma (1990), Blackburn and Cipriani (2002), Chakraborty (2004), Chakraborty and Das (2004) explore some aspects of the dynamic characteristics of longevity and per capita income, the relationship with fully endogenous longevity is not analysed systematically. This paper aims to clarify the relationship between life expectancy and economic growth using a two period overlapping generations model similar in form to that of Blackburn and Cipriani (2002), Chakraborty (2004) and Chakraborty and Das (2004). However, the particular focus of this paper is the interaction between health investment and schooling investment and their roles in economic development.

This paper is structured as follows. In the next section a brief review of the literature on economic growth using exogenous life expectancy is given, followed by a more detailed review of two of the key papers using endogenous life expectancy. The model used in this paper is then developed, and the first order conditions lead to analysis of the corner solutions in health and schooling investment, and the Euler equations are analysed in the the context of a full interior solution with differing degrees of risk aversion. The payoff between health and schooling investment is analysed by simulating the model. Concluding comments are then made.

2 Models of longevity and growth

2.1 Exogenous longevity

Theoretical models proposed by authors such as Zhang et al. (2001), de la Croix and Licandro (1999), Rosenzweig (1990), Kalemli-Ozcan et al. (2000), Boucekkine et al. (1999), Kalemli-Ozcan (2002) construct an overlapping generations model and demonstrate that an exogenous increase in life expectancy lengthens the time horizon over which returns to human capital investment can be earned, thereby influencing an increase in the latter. The enhanced accumulation of human capital drives endogenous growth. However, their analysis suffers from two limitations: the assumed direction of causality and neglect of the role of costly investment in health and thus life expectancy.

In models of exogenous longevity the unidirectional causal link between life expectancy and income severely restricts the scope for analysis and policy application. The imposition of this restrictive assumption regarding the causal direction leads to the conclusion that a universal increase in life expectancy, by exogenously determined means, will boost economic growth in all countries from low to high per capita income.

Here we argue that life expectancy may increase as a result of deliberate and costly investment. Unlike other exogenous variables in these models, for example, the rate of time preference, life expectancy is not a feature of an individual's preferences, but is achieved by way of the allocation of resources to goods and services that will enhance the individual's health and probability of survival. Investing in health increases the probability of gaining utility over second period consumption. The source of these resources can be government provision or private investment. Either avenue requires individuals to forego current consumption in favour of investment in health that will enhance the probability of survival. To model investment in health and longevity, it must be a choice variable with resource costs.

Incorporation of the opportunity cost of investing in health (in addition to that of schooling) into the lifetime consumption decision does not lead to the same policy conclusion as models of exogenous longevity, rather at low levels of income zero investment in health can be optimal, and actively increasing investment in life extension may be sub-optimal. The model developed in this paper explores in greater detail the interaction between life expectancy and economic growth by allowing for the endogeneity of life expectancy and health is a choice variable of the expected utility maximising individual. The individual makes the choice over investing in schooling to increase second period consumption (the risky asset) or increasing the probability of achieving the risky asset through investment in health.

2.2 Endogenous Longevity

Models of endogenous longevity are currently in their infancy, although they are increasingly the focus of analysis relating demographic variables with economic growth. Seminal works by Grossman (1972) and the more refined paper by Ehrlich and Chuma (1990) introduce the concept of the demand for health and longevity. The rigor of the Ehrlich and Chuma (1990) paper is impressive, and the development of the value of health capital, that is lacking in models of exogenous longevity, is articulated convincingly. The opportunity cost of foregone consumption for the provision of resources for investment in health is identified in these two papers. Ehrlich and Chuma (1990) only briefly discuss the relationship between endogenous longevity and economic growth (page 780), the focus of their work is the demand for health rather than the dynamic economic growth effects.

The modelling framework adopted in the current paper extends works by Blackburn and Cipriani (2002) and Chakraborty (2004) who attempt to model endogenous longevity and economic growth. Both models feature endogenous growth driven by human capital accumulation; Blackburn and Cipriani (2002) augment their model with variable labour supply and Chakraborty (2004) opts for augmentation with physical capital accumulation. The Blackburn and Cipriani (2002) paper is used in developing the core framework of this paper where individuals are both consumers and producers. In drawing on their foundation, the current model incorporates their production functions for output and human capital accumulation.

Blackburn and Cipriani (2002) and Chakraborty (2004) endogenise longevity in different ways. Both are discrete time models, and analyse the endogeneity of the probability of survival into the final period of life. In Blackburn and Cipriani (2002), the life expectancy of a child is pre-determined by the choice of human capital accumulation of their parents. Chakraborty (2004) endogenises the probability of survival through an optimally chosen health tax. The concept of an optimally chosen proportion of income devoted to health as in Chakraborty (2004), is adopted in this paper, although it is treated as a matter of individual choice rather than public choice.

Blackburn and Cipriani (2002) find in their model of development with dynamically endogenous life expectancy, that a higher probability of survival is associated with a higher steady state level of human capital, and potentially endogenous growth (Blackburn and Cipriani (2002), p.194). The mechanism by which a higher life expectancy is achieved is through a higher level of human capital investment by the parent, not of the current utility maximising individual. Chakraborty (2004) also finds that a higher life expectancy is associated with a faster rate of convergence from the same initial conditions (other than life expectancy) as the steady state level of income increases with the increase in life expectancy and the economy is now further away from the steady state and hence transitional dynamics dictate that the growth rate will be faster.

A feature of these two papers which endogenise life expectancy is the existence of poverty traps, or economic stagnation, can be explained. Blackburn and Cipriani (2002) show that if the initial level of human capital is below the threshold level then the economy will stagnate and as life expectancy is dependent upon the level of development of the economy, persistent low income translates to persistent low life expectancy. Economies that are above the threshold of initial human capital stock experience endogenous growth and rising life expectancy. Chakraborty (2004) explains that in their model, development traps are possible, and in high mortality economies the incentive to invest (in physical or human capital) is low and hence growth stagnates.

Despite the merits of these two papers of endogenous longevity and economic growth, they have serious drawbacks, and there is scope to develop the modeling. The Blackburn and Cipriani (2002) paper's use of parents' human capital implies a dynamically endogenous probability of survival; but life expectancy is not a control variable of the utility maximising agent. As longevity is not a choice variable of the agent, parents' utility function remains additively separable (Blackburn and Cipriani (2002) equation (1)) and a closed form solution is proposed. Ignoring the opportunity cost of extending life expectancy affords a modelling simplification, but restricts the value added of this model over models of exogenous longevity. The differences between exogenous and endogenous models of longevity are driven by the recognition of this opportunity cost, and skirting this important aspect detracts from the value of endogenising longevity.

Chakraborty (2004) does address the opportunity cost of investment in health through the health tax chosen optimally by the utility maximising individual (see Appendix B, Chakraborty (2004)). The fundamental problem with the analysis of Chakraborty (2004) is that he does not utilise the endogeneity of the survival probability and the associated opportunity cost to address key conclusions; rather, he conducts the analysis assuming a fixed health tax. Yet, in the development of Proposition 1 (see page 126), the discussion reverts to endogenous longevity even though the preceding analysis was conducted assuming exogenous longevity. On closer inspection, this proposition pertaining to the mechanism by which differences in productivity drive cross-country differences in life expectancy can be attained with exogenous longevity - endogeneity of longevity is not required as is asserted by Chakraborty (2004). Appendix 1 of this paper details this criticism.

Each of the above models explores the relationship between endogenous longevity and economic growth. The relationship between life expectancy, human capital accumulation and endogenous life expectancy feature in all of these models. The current model, however, offers a comparison of human capital investment and health capital investment and the role each plays in their contribution to economic growth. Moreover, decisions over these investments consider the individuals' attitude towards risk, as decisions are made under uncertainty.

3 A model of endogenous longevity and economic growth

In a two period overlapping generations model, the fertility rate is predetermined with one child born to each individual at the end of period one, and the individual will maximise their lifetime expected utility over consumption in the current and future periods. In the first period of life, the individual makes decisions over labour supply and investment in schooling, the latter contributes to the accumulation of the stock of human captial. Consumption in this period is increasing in labour supply, but investment in schooling will increase second period consumption. This investment, however, is considered risky as the individual will only survive into the second period probabilistically depending on their health status. If the individual survives into the second period the individual supplies labour and is awarded depending on their accumulated level of human capital. Human capital accumulation is the engine of growth, and an economy can stagnate if investment in schooling is non-existent or insufficient.

The probability of survival into the second period is endogenised. In the first period, individuals not only make decisions over how much to invest in the risky investment (schooling), but also how much to invest in health which determines the probability of attaining the risky asset (period two consumption). From a situation where investment in schooling and health is zero, which investment the individual will make first (health or schooling) will depend on the individuals degree of risk aversion. For example, as the simulation results show, an individual who is highly risk averse will first invest in health to increase the probability of a payoff, before taking the gamble by investing in schooling.

The model is defined as follows and elaborates on two fronts from the model of exogenous longevity above: firstly, and obviously, life expectancy is endogenous, and secondly, schooling is split into a compulsory and voluntary level. The reason for the latter is to show that an individual's willingness to voluntarily invest in health, or schooling, will depend on the degree of risk aversion. Without the compulsory schooling component this cannot be determined as the marginal utility of schooling investment is infinite at $schooling = 0$ if compulsory schooling is not in the model.

To summarise, the model is a two period overlapping generations model of endogenous survival probability, and exogenous fertility, where economic growth is driven by human capital accumulation. The individual aims to maximise their expected lifetime utility over current and future consumption, and in the first period of life, the individual has the choice over current consumption, investment in schooling to raise second period consumption, and investment in health to increase survival probability[1]. In the second period of life, the individual consumes all that is produced. Individuals then make the choice of investing in neither health or schooling, one of these or both, depending on their degree of risk aversion as income accumulates.

3.1 ***Expected Utility***

Individuals born in period t maximise their expected lifetime utility over consumption in the first period of life, c_t^t, and that in the second period, c_{t+1}^t. The expected utility of death is zero.

$$U_t^t = u\left(c_t^t\right) + \pi_{t+1} u\left(c_{t+1}^t\right) \tag{1}$$

It is assumed that utility is increasing and concave in consumption, $u'(c) > 0$ and $u''(c) < 0$.

The probability of survival, π_{t+1}, acts as an effective discount rate, and is a choice variable (via health investment, h_t^t) within the individual's maximisation problem. The rate of time preference is set at zero in this example, and as this exogenous parameter plays no role in the analysis, this assumption has no limiting consequences. Although the rate of time preference is set to zero, the

[1]The terms survival probability, longevity and life expectancy are used interchangeably.

individual is not strictly indifferent between consumption in the first and second periods as survival through period two is probabilistic, $\pi_{t+1} \in (0,1)$, the felicity for consumption in period one carries a greater weight in its contribution to lifetime utility if c_t^t and c_{t+1}^t are equal. Survival through the first period is certain and hence the notation of $\pi_t = 1$ could be included. To this end, this is a model of adult mortality.

A felicity function with a constant relative risk aversion has the properties required of the expected utility function in equation (1), and takes the form,

$$U_t^t = \frac{(c_t^t)^{1-\gamma}}{1-\gamma} + \pi(h_t, \bar{z})\frac{(c_{t+1}^t)^{1-\gamma}}{1-\gamma}, \gamma \in (0,1) \tag{2}$$

Where $\gamma = \frac{1}{\sigma}$. Being a model with uncertainty, the familiar constant intertemporal elasticity of substitution is better termed as a constant relative risk aversion in c_t^t and c_{t+1}^t[2], where σ is the measure of risk aversion. An individual with constant relative risk aversion (as opposed to increasing or decreasing) will have no greater willingness to except a gamble (invest in period two consumption) with an increase in wealth. The higher is the relative risk aversion (the higher is γ, $\sigma \to 0$), the more risk averse is the individual and they will require a greater risk premium to encourage investment in second period consumption where the length of time over which returns to such investment is uncertain. A value of $\gamma > 1$ will yield a negative felicity function in consumption, hence $\gamma < 1$ is assumed so that consumption has the intuitive effect of positive expected utility even at very low levels of consumption[3]. Note that $\gamma = 0$ gives a

[2]The measure of constant relative risk aversion, $r_R(c_{t+i}, u) = \frac{c_{t+i}u''(c)}{u'(c)} = -\gamma$ is constant and not influenced by the endogenous probability of survival.

[3]In this model of uncertainty, expected utility is interpreted as cardinal utility and not ordinal utility, hence the value of utility matters. Moreover, in this OLG model the implicit expected utility of death is zero, so when expected utility of consumption is negative then the individual will always prefer dying to living no matter how high consumption is if $\gamma > 1$ and

risk neutral person, and as $\gamma \to 1$ the degree of risk aversion increases. $\gamma = 1$ will give a felicity function logarithmic in consumption.

3.2 *Survival Function*

The probability of survival, π_{t+1}, is determined in part by the level of investment in health as chosen by the individual in period one, h_t, and in part by factors historically determined or existing public health measures, $\bar{z}$. The survival function has the following properties.

$$\begin{aligned}
&\pi_{t+1} = \pi\left(h_t, \bar{z}\right);\ \pi_{t+1}\left(0, \bar{z}\right) > 0;\ \pi_{t+1}(\infty, \infty) \leq 1 \\
&\pi_h\left(h_t, \bar{z}\right) > 0;\ \pi_z\left(h_t, \bar{z}\right) > 0;\ \pi_{hh}\left(h_t, \bar{z}\right) < 0; \\
&\pi_h(0, \bar{z}) \in (0, \infty);\ \pi_h(0, 0) = \infty
\end{aligned} \tag{3}$$

There is a baseline probability of survival that is a function of an exogenously determined level of health, $\bar{z}$. Investment in health could be directed towards public goods such as closed sewerage system, access to clean water or a public hospital system, or private health benefits such as private medical insurance, healthy food, and exercise, all of which are assumed to increase life expectancy beyond the baseline level. Private health investment, h_t, contributes positively to the probability of survival through the second period. An individual lives with certainty through the first period, and with probability π_{t+1} lives through the second period, otherwise the individual dies with probability $1-\pi_{t+1}$ at the end of the first period[4]. Disinvestment in health, for example smoking cigarettes,

in this situation strictly positive investment in health and schooling will never eventuate.

[4]In the two period OLG context we could also interpret the probability such that the individual expects to live for $1 + \pi$ periods, however, this interpretation does not lend itself to applications of stochastic dynamic programming - the concept behind the simulations.

is not modelled here, but zero investment in health is a feasible decision of the utility maximising individual.

3.3 *Human capital accumulation and transmission*

In the first period of life, individuals divide their time between compulsory schooling, $\bar{s}$, voluntary schooling, s_t, and labour supply. Schooling augments inherited human capital in a multiplicative fashion, reflecting the hypothesis that the higher the level of initial human capital, the greater the benefit of schooling. Thus a person born in period t enters the second period of life with human capital, e_{t+1}^t, given by

$$e_{t+1}^t = Be_t^t \left(s_t + \bar{s}\right), \, B > 1 \tag{4}$$

Human capital is linked across generations by the assumption that children are not only born with a biological level of human capital, $\bar{e}$, but they also inherit[5] their parents' acquired level of human capital

$$e_t^t = e_t^{t-1} + \bar{e} \tag{5}$$

Equations (4) and (5) together describe the evolution of human capital as a first order difference equation which can be rewritten as,

$$e_{t+1}^t = B(\bar{s} + s_t)e_t^{t-1} + B\bar{e}(\bar{s} + s_t) \tag{6}$$

Being a linear autonomous fist order difference equation, equation (6) solves

[5]This inheritance might be acquired through informal education in the home which is treated as costless for the purpose of this paper.

to be,

$$e_t = \bar{e}\{\frac{1 - 2B(s_t + \bar{s})}{1 - B(s_t + \bar{s})}\}\{B(s_t + \bar{s})\}^t + \frac{B\bar{e}(s_t + \bar{s})}{1 - B(s_t + \bar{s})} \tag{7}$$

For endogenous growth, e_t must grow exponentially. For this to be such, then this requires restrictions on the parameter values.

The economy will stagnate to the steady state of human capital if, $B(s_t+\bar{s}) < 1$, in which case the growth component of equation (7) will approach zero in the limit,

$$\lim t \to \infty \ \{B(s_t + \bar{s})\}^t = 0 \text{ if } B(s_t + \bar{s}) < 1 \tag{8}$$

For endogenous growth then this requires,

$$B(s_t + \bar{s}) > 1$$
$$\text{which then ensures, } \bar{e}\{\frac{1 - 2B(s_t + \bar{s})}{1 - B(s_t + \bar{s})}\} > 0 \tag{9}$$

High productivity of human capital in human capital production, B, must be coupled with a high rate of schooling investment (voluntary or compulsory) to ensure that this holds[6]

3.4 *Production function*

Output in each period, y_T^t where $T = t, t+1$, is a multiplicative function of inherited human capital, e_T^t, labour supply, l_T^t, and a productivity parameter,

[6]The simulation results conducted later in the paper indicate that it is the low productivity of human capital in human capital production that causes the poverty traps, not low compulsory schooling levels. In running the simulations with parameter values as those defined later, but with $B = 9$, $\bar{s} = 0.1$ then stagnation does not occur, but with $B = 3$ and $\bar{s} = 0.3$ the economy will be caught in a poverty trap even though in both cases $B < \frac{1}{\bar{s}}$.

A. Per capita income is essentially a function of productive labour.

$$y_T^t = Ae_T^t l_T^t, \text{ where T=t, t+1} \tag{10}$$

This is the same form adopted by Blackburn and Cipriani (2002). There is no physical capital, and savings is in the form of schooling and it is the avenue by which income is transferred from period one to period two. Increasing the level of schooling will require the individual to supply less labour in period one which will lower y_t^t, but increase the stock of human capital taken into the second period and subsequently increase y_{t+1}^t.

3.5 *Budget constraints*

The individual is constrained by time and income. In the first period, an individual must divide their unit of time between labour supply (l_t^t), compulsory schooling ($\bar{s}$), and voluntary schooling (s_t).

$$l_t^t \leq 1 - \bar{s} - s_t \tag{11}$$

In the first period, income is divided between consumption and investment in health. In the second period, however, consumption of goods and services is the only avenue of expenditure.

$$c_t^t \leq y_t^t - h_t \tag{12}$$

$$c_{t+1}^t \leq y_{t+1}^t \tag{13}$$

However, non-satiation of expected utility in consumption and the assumption of no leisure imply that the constraints hold with equality.

3.6 *Problem of the agent*

Given that equations (12) and (13) can be written as equalities, the problem of the utility maximising individual can be reduced to a problem over (s_t, h_t) [7].

$$\text{Maximise } U_t^t\left(s_t, h_t\right) = u\left(Ae_t^t(1-\bar{s}-s_t)-h_t\right) + \pi\left(h_t, \bar{z}\right) u\left(ABe_t^t\left(\bar{s}+s_t^t\right)\right) \tag{14}$$

$$\text{Subject to } s_t \geq 0;\ \ h_t \geq 0 \tag{15}$$

The Lagrangian is then expressed as

$$L\left(s_t, h_t, \lambda, \mu\right) = u(Ae_t^t(1-\bar{s}-s_t)-h_t) + \pi(h_t, \bar{z})u(ABe_t^t(\bar{s}+s_t^t)) + \lambda s_t + \mu h_t \tag{16}$$

The first order necessary conditions are

$$\frac{\partial L}{\partial s_t} \Rightarrow u'(c_t^t)Ae_t^t = \pi(h_t, \bar{z})u'(c_{t+1}^t)ABe_t^t + \lambda \tag{17}$$

$$\frac{\partial L}{\partial h_t} \Rightarrow u'(c_t^t) = \pi_h(h_t, \bar{z})u(c_{t+1}^t) + \mu \tag{18}$$

$$\lambda s_t = 0;\ s_t \geq 0;\ \lambda \geq 0 \tag{19}$$

$$\mu h_t = 0;\ h_t \geq 0;\ \mu \geq 0 \tag{20}$$

[7]As labour supply is normalised to one, then schooling investment which represents a proportion of time is bound between zero and one, whereas investment in health is represented in the level and with the non-negativity constraint then health is bound between zero and the level of first period income.

The first order conditions and Euler equations are used to conduct analysis as a closed form solution cannot be deduced with this utility function which is not additively separable. Threshold incomes at which schooling and health become positive, and the payoff between health and schooling are anaylsed in the following sections.

3.7 *Threshold Incomes for Strictly Positive Investment in Schooling*

Recalling that the utility function is expressed in a functional form with constant relative risk aversion in consumption, and hence the first order condition in (17) can be expressed as,

$$\frac{\pi(h_t, \bar{z})ABe_t}{(ABe_t(s_t + \bar{s}))^\gamma} + \lambda = \frac{Ae_t}{(Ae_t(1 - s_t - \bar{s}) - h_t)^\gamma} \tag{21}$$

At the threshold level of income where an individual will switch from zero to positive investment in schooling, then the strict inequality constraint is binding when schooling investment is zero and so too is the complementary slackness variable, λ[8]. This gives,

$$\frac{\pi(h_t, \bar{z})ABe_t}{(ABe_t\bar{s})^\gamma} = \frac{Ae_t}{(Ae_t(1 - \bar{s}) - h_t)^\gamma} \tag{22}$$

Rearranging this, the first period income at which this switch from zero to

[8]To satisfy the Kuhn Tucker conditions of constrained optimisation with inequalities, if the constraint is binding so that it holds with an equality, in this case $s_t = 0$, then the multiplier, λ, must be ≥ 0. When the constraint is not binding, and $s_t > 0$, then the multiplier must be zero, $\lambda = 0$. Hence at the threshold where the constraint is binding with zero investment in schooling, and the indifference curve associated with the expected utility function is tangent to the constraint at $s_t = 0$ and the multiplier is also zero, $\lambda = 0$. Source: Simon and Blume (note, this is yet to be adequately cited)

positive investment in schooling can be determined[9],

$$Ae_t(1-\bar{s}) = \frac{ABe_t\bar{s}}{\pi(h_t,\bar{z})^{\frac{1}{\gamma}}B^{\frac{1}{\gamma}}} + h_t \tag{23}$$

This level of income represents the level of income required to encourage strictly positive investment in schooling. Given that schooling investment is a risky investment, the payoff of higher second period consumption is not certain as survival through the second period of life is probabilistic, then we would expect that an individual with a higher the degree of risk aversion will require a higher threshold income at which investment in the risky asset becomes strictly positive. This is indeed confirmed by the simulation results presented later in the paper.

Given the term for risk aversion γ, is in the exponent, taking the log of both sides and then taking the derivative with respect to γ gives,

$$\frac{\partial \ln\{Ae_t(1-\bar{s}) - h_t\}}{\partial\gamma} = \frac{1}{\gamma^2}\ln\{\pi(h_t,\bar{z})B\} \tag{24}$$

This will be strictly positive, if $\pi(h_t,\bar{z}) > \frac{1}{B}$ and thus the level of income required before schooling is strictly positive is increasing in the degree of risk aversion. This positive relationship between the threshold first period income and the degree of risk aversion can be examined by looking at three cases,

Case 1: $\gamma = 1$, the highest degree of risk aversion in this model (log utility), the threshold income is thus,

$$Ae_t(1-\bar{s}) = \frac{ABe_t\bar{s}}{\pi(h_t,\bar{z})B} + h_t \tag{25}$$

[9] A corner solution in health is not assumed as this imposes zero investment in health when schooling switches when indeed there is not reason for it not to be positive as the simulation results indicate.

Case 2: $\gamma \in (0, 1)$ risk averse,

$$Ae_t(1-\bar{s}) = \frac{ABe_t\bar{s}}{\pi(h_t,\bar{z})^{\frac{1}{\gamma}}B^{\frac{1}{\gamma}}} + h_t \tag{26}$$

Case 3: $\gamma = 0$, risk neutral,

$$Ae_t(1-\bar{s}) = \frac{ABe_t\bar{s}}{\pi(h_t,\bar{z})^{\frac{1}{0}}B^{\frac{1}{0}}} + h_t = h_t \tag{27}$$

With,

$$h_t < \frac{ABe_t\bar{s}}{\pi(h_t,\bar{z})^{\frac{1}{\gamma}}B^{\frac{1}{\gamma}}} + h_t < \frac{ABe_t\bar{s}}{\pi(h_t,\bar{z})^{\frac{1}{0}}B^{\frac{1}{0}}} + h_t = h_t \tag{28}$$

Which requires, $\pi(h_t,\bar{z}) > \frac{1}{B}$ as the derivative above also shows.

Thus, the level of income that is required at the threshold of an individual switching from zero to positive investment in schooling is increasing in the degree of risk aversion. Column 4 of Table 1 of the simulation results presented in the appendix confirm this.

3.8 *Threshold Incomes for Strictly Positive Investment in Health*

Turning now to the first order condition with respect to health, equation (18), and using the functional form of constant relative risk aversion in consumption, then the first order condition can be expressed as,

$$\pi_h(h_t,\bar{z})\frac{(ABe_t(\bar{s}+s_t))^{1-\gamma}}{1-\gamma} + \mu = (Ae_t(1-\bar{s}-s_t)-h_t)^{-\gamma} \tag{29}$$

Again, the threshold at which investment in health switches from zero to positive is health investment is zero, but the strict inequality of the complementary slackness condition is binding and μ is also zero. Using this condition and rearranging and the level of income at which investment in health switches from zero to positive is,

$$Ae_t(1-\bar{s}-s_t) = \{\frac{1-\gamma}{\pi_h(0,\bar{z})(ABe_t(\bar{s}+s_t))^{1-\gamma}}\}^{\frac{1}{\gamma}} \tag{30}$$

Comparative static shows that,

$$\frac{\partial \ln(Ae_t(1-s_t-\bar{s}))}{\partial\gamma} = \frac{1}{\gamma^2} ln((1-\gamma)\pi_h(0,\bar{z})ABe_t(s_t+\bar{s})) \tag{31}$$

where

$$\begin{aligned} \frac{\partial}{\partial\gamma} &< 0 \text{ if } (1-\gamma)\pi_h(0,\bar{z})ABe_t(s_t+\bar{s}) < 1 \\ \frac{\partial}{\partial\gamma} &> 0 \text{ if } (1-\gamma)\pi_h(0,\bar{z})ABe_t(s_t+\bar{s}) > 1 \end{aligned} \tag{32}$$

Where $\frac{\partial}{\partial\gamma} > 0$ implies that the income required before health investment is strictly positive is increasing in the degree of risk aversion. Given the different degrees of risk aversion are expressed in terms of the value γ takes on, then looking at three cases helps us understand how attitudes towards health investment differ according to the degree of risk aversion.

Case 1: $\gamma = 1$, log utility and the highest degree of risk aversion in this study.

$$Ae_t(1-\bar{s}-s_t) = \frac{1}{\pi_h(0,\bar{z})\ln(ABe_t(s_t+\bar{s}))} \tag{33}$$

Case 2: $\gamma \in (0,1)$, the individual is risk averse.

$$Ae_t(1-\bar{s}-s_t) = \{\frac{1-\gamma}{\pi_h(0,\bar{z})(ABe_t(\bar{s}+s_t))^{1-\gamma}}\}^{\frac{1}{\gamma}} \tag{34}$$

Case 3: $\gamma = 0$, the individual is risk neutral.

$$\begin{aligned} Ae_t(1-\bar{s}-s_t) &= 0 \text{ if } 1-\gamma < \pi_h(0,\bar{z})(ABe_t(\bar{s}+s_t)) \\ &= \infty \text{ if } 1-\gamma > \pi_h(0,\bar{z})(ABe_t(\bar{s}+s_t)) \end{aligned} \tag{35}$$

Looking at these three cases then we can see that the income required to encourage strictly positive investment in health is increasing in the degree of risk aversion if $1-\gamma < \pi_h(0,\bar{z})(ABe_t(\bar{s}+s_t))$. Simulation results presented later in the paper help to understand this relationship as the threshold income is not clearly increasing or decreasing in the degree of risk aversion as column 2 of Table 1 in the appendix shows.

This analysis tells us the threshold levels of income for switching to strictly positive investment in health and/or schooling. The nature of the two types of investment are quite different, both are costly in terms of forgoing first period consumption but schooling has the effect of increasing the value of period two consumption, while health has the effect of increasing the probability of achieving period two consumption.

3.9 *Optimal Payoff Between Health and Schooling*

The Euler equations show the optimal payoff between the two choice variables, schooling and health. With a full interior solution, the first order conditions can be written as,

$$\text{wrt s: } \frac{\pi(h_t, \bar{z}) ABe_t}{(ABe_t(s_t + \bar{s})^{\gamma}} = \frac{Ae_t}{(Ae_t(1 - s_t - \bar{s}) - h_t)^{\gamma}} \tag{36}$$

$$\text{wrt h: } \frac{\pi_h(h_t, \bar{z})(ABe_t(s_t + \bar{s}))^{1-\gamma}}{1-\gamma} = \frac{1}{(Ae_t(1 - s_t - \bar{s}) - h_t)^{\gamma}} \tag{37}$$

Using these two first order conditions, the Euler equation can be deduced as,

$$\frac{1-\gamma}{Ae_t(s_t + \bar{s})} = \frac{\pi_h(h_t, \bar{z})}{\pi(h_t, \bar{z})} \tag{38}$$

Totally differentiating this Euler equation to determine the relationship between health and schooling investment, and we get,

$$\frac{-(1-\gamma)Ae_t}{(Ae_t(s_t + \bar{s}))^2} ds = \frac{\pi_{hh}(h_t, \bar{z})\pi(h_t, \bar{z}) - \pi_h(h_t, \bar{z})^2}{\pi(h_t, \bar{z})^2} dh \tag{39}$$

Recalling the properties of the survival function, it can be seen that both sides are negative, indicating the health and schooling change in the same direction and thus the calculation of the elasticity of health with respect to schooling (or vise versa) would show that the two investments are complements.

3.10 ***Preliminary Simulation Results***

Thus far we have shown that from the first order conditions that the level of income required to encourage strictly positive investment in schooling is increasing in the level of risk aversion, and also for health investment under given parameter restrictions. Moreover, the Euler equation shows that investment in health and schooling are complements. Simulation results are used to show the dynamic process and how investment decisions over health and schooling are made as human capital accumulates over the generations.

The following demonstrates the conditions required to determine which investment comes first as income increases with human capital accumulation (which is always possible even when voluntary investment in schooling is zero as there is an element of compulsory schooling). In conducting the simulations, assumptions over parameter values, initial conditions, and the functional form of the survival function need to be made. Given that the production functions are based on the paper by Blackburn and Cipriani (2002), where possible, parameter values used by these authors are adopted in this paper. The functional form of the survival function used in this paper is also an adaptation of Blackburn and Cipriani (2002). In this case, $A = 1$, $B = 9$, $\bar{e} = 0.1$, $\underline{\pi} = 0.3$, $\bar{\pi} = 0.95$, $\Phi = 0.01$. In addition to these parameter value assumptions, we assume, $\phi = 0.5$, and compulsory schooling is $\bar{s} = 0.3$. The functional form of the probability of survival is,

$$\pi(h_t, \bar{z}) = \frac{\underline{\pi} + \bar{\pi}\Phi(h_t)^{\phi}}{1 + \Phi(h_t)^{\phi}} \tag{40}$$

With these parameter values, simulations are run over fifty generations[10]. Income accumulates each successive generation according to the equation for human capital, with $B > \frac{1}{\bar{s}}$, the condition for endogenous growth is satisfied (according to requirements for exponential accumulation in equation (7)) even without voluntary investment in schooling. Hence, with the above chosen parameter values, a poverty trap is not a threat ($B = 9 > \frac{1}{\bar{s}} = \frac{10}{3}$). In conducting the simulations, different levels of risk aversion are assumed, $\gamma = [0, 0.1, 0.2, ..., 0.9]$,

[10]This is essentially a truncated infinite horizon stochastic dynamic programming problem

and each time the simulations are run the generation in which health and schooling become strictly positive is observed, and the associated level of first period income and expected lifetime utility are also noted. The ordering of these investments differs as the degree of risk aversion changes, so too does the level of income required to encourage investment in either health or schooling.

Figure 1 in the appendix illustrates the proportion of time invested in schooling decreases as risk aversion increases. At low levels of risk aversion, ($\gamma \rightarrow 0$), an individual will invest strongly in schooling thereby taking resources from first period income but then earn a skilled wage in the second period. This decision to invest in the risky asset (second period consumption) stems from the fact that a risk neutral individual (where $\gamma = 0$) will base their decisions under uncertainty on expected value, and in this case expected value and expected utility are the same.

Not only is the rate of investment in schooling decreasing in the degree of risk aversion, but the generation at which this investment initially occurs is increasing in the degree of risk aversion. This is representative of the fact that the higher the degree of risk aversion, the greater the first period income must be before an individual is willing to switch from zero to strictly positive investment in schooling.

Figure 2 shows the level of health investment is decreasing in the degree of risk aversion. Health investment has the effect of increasing life expectancy, thereby increasing the probability of achieving second period income. But as an individual becomes more risk averse, their willingness to invest in health will decline, for as with schooling returns to health investment will not be had if the individual dies at the end of the first period.

The willingness for the individual to invest in health and schooling can be thought of in a risk analysis setting. Individuals who are risk averse will base their decisions on expected utility, not expected value as a risk neutral person would, and to encourage an individual to take a gamble then they must be awarded an appropriate risk premium (an amount the individual receives with certainty when taking the gamble) to do so. Comparing two degrees of risk aversion, say high and low, an individual with a high degree of risk aversion will require a higher risk premium for the same gamble as an individual with a lower degree of risk aversion. Alternatively, we can interpret this comparison as for a given initial income an individual with the higher degree of risk aversion will gamble less than the individual with a lower degree or risk aversion. So, for a given level of income, the amount an individual is willing to gamble falls as the degree or risk aversion increases. This results is observed in the simulation results presented in Figures 1 and 2.

Table one shows that as the degree of risk aversion increases so too does the threshold level of income required before an individual will switch to strictly positive investment in schooling. With the degree of risk aversion $\gamma \in [0, 0.2]$ schooling and health investment switch to strictly positive simultaneously at the beginning of the first generation. With mid levels of risk aversion, $\gamma \in [0.3, 0.7]$, investment in schooling precedes investment in health. For example with $\gamma = 0.6$ schooling investment occurs from the first generation, but health investment does not switch to strictly positive until the forth generation. At high degrees of risk aversion, $\gamma \in [0.8, 1.0]$, investment in schooling occurs after investment in health, as an example when, $\gamma = 0.8$ initial investment in health occurs at the third generation but investment in schooling does not commence until the

11^{th} generation. The ordering of the investments depends on the return each investment offers at different degrees of risk aversion.

The ordering of the investments changes as the degree of risk aversion changes as each investment plays a different role in the decision making under uncertainty. Investing in schooling has the effect of increasing the expected value of the risky asset, and investment in health increases the probability of achieving the risky asset. When risk aversion is high, an individual will invest in health to first increase the probability of achieving the risky asset before they take the gamble on the risky asset by investing in schooling.

The primary achievement of these preliminary simulation results is to show that the interaction between the optimal health and schooling investments is dependent upon the degree of risk aversion the representative agent faces. The results prove to be quite intuitive and provide direction for further work in terms of formalising the latter section of this paper to outline a stochastic dynamic programming model, undertake formal risk analysis, calculate risk premiums, and returns on investments at different degrees of risk aversion and income levels.

4 Conclusion

Prior to this paper, models of endogenous longevity did not explore the dynamic relationship between investment in health, life expectancy, schooling and economic growth. Using a two period overlapping generations model where both schooling and health investment are choice variables of the individual, interactions between health and schooling can be analysed more thoroughly than in models of exogenous longevity. Both health and schooling are costly in terms of foregone first period consumption, but the payoff is different for each investment. Schooling investment has the effect of increasing the return to labour in the second period thus increasing second period consumption. However, the individual only lives through the second period probabilistically and this probability is determined by the level of investment in health. Investment in schooling is thus risky, and health investment has the effect of increasing the probability of achieving the payoff of this risky investment in schooling.

In further exploring the relationship between health and schooling and their effects on economic growth, the first order conditions reveal that threshold levels of income exist where schooling and health switch to strictly positive. For schooling investment, the threshold level of income is increasing the the degree of risk aversion. As risk aversion increases the individual must secure a higher income with certainty before they are willing to take the gamble of investing in schooling. The threshold income for health investment does not monotonically increase with the degree of risk aversion.

The Euler equations show that health and schooling investments are complements. In this model there is no quality/quanity trade off of length of life and higher consumption. The two investments move together.

The first order conditions showed that thresholds exist, but we were not able to decipher from these alone when investment in health and schooling would occur relative to the other as income accumulates. Simulation results showed that when the degree of risk aversion is low then the individual will invest in schooling first and then as income further accumulates then invest in health. For individuals who are more risk averse, they will invest in health first to increase the probability of achieving the risky asset before investing in schooling.

The key contribution of this model of endogenous life expectancy over models of exogenous life expectancy is to show that the relationship between health and schooling is highly interactive and dependent upon the degree of risk aversion. The policy prescription of increasing life expectancy to boost economic growth does not hold when considering the endogeneity of both health and schooling as it does in models of exogenous longevity.

This model is just the beginning of a number of extensions that are possible. In the discrete time model, fertility timing and quantity as in Blackburn and Cipriani (2002) can be added, accounting for different levels of schooling (primary, secondary and tertiary), and incorporating child mortality are just some of the possible avenues of further research. Moreover, this model provides a theoretical foundation for empirical models that look at not just the labour productivity effect of health on economic growth but also the incentive effects of health investment and life expectancy on schooling investment and thus economic growth. This empirical paper is part of work we are currently undertaking.

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5 Appendices

5.1 Appendix 1

Chakraborty (2004) asserts that using his specified Equations (1), (2) and (6) it can be shown that differences in productivity, A, lead to differences in the capital to output ratio, because for any given capital stock, a lower A will reduce longevity through lower income and health investment. The endogeneity of life expectancy is implied through a chosen health tax, τ_t. But it is not the endogeneity of life expectancy that is driving differences in productivity as *Proposition 1* of Chakraborty (2004) claims.

From Equation (1) in Chakraborty (2004):

$$\phi_t = \phi(h_t) \tag{41}$$

From Equation (2) in Chakraborty (2004):

$$(h_t) = g(\tau_t, w_t) = \tau_t w_t \tag{42}$$

From Equation (6) in Chakraborty (2004):

$$w_t = w(k_t) = (1 - \alpha) A k_t^{\alpha} \tag{43}$$

To identify the relationship between productivity and life expectancy from these three equations, the first derivative of each is take with respect to the shared arguments.

From equation (41),

$$\frac{\partial w_t}{\partial A} > 0 \tag{44}$$

From equation (42),

$$\frac{\partial h_t}{\partial w_t} > 0 \tag{45}$$

From equation (43),

$$\frac{\partial \phi_t}{\partial h_t} > 0 \tag{46}$$

These fist derivatives show that if productivity falls, the wage rate will fall, which in turn lowers health and life expectancy. Differences in life expectancy implied by a change in productivity are generated by changes in the wage rate, not a variable health tax rate. It is not the endogeneity of the health tax that is driving this relationship between productivity and life expectancy as asserted by Chakraborty (2004), rather the proposition (page 126) can be achieved with endogenous or exogenous health tax.

5.2 Appendix 2: Selected Simulation Results

Table 1: Threshold incomes, expected utility and generation at which investment switches from zero to strictly positive.

γ	y_t^h	U_t^h	y_t^s	U_t^s	Gen. t^h	Gen. t^s
0	0.003	0.265	0.003	0.265	1	1
0.1	0.003	0.301	0.003	0.301	1	1
0.2	0.006	0.349	0.006	0.349	1	1
0.3	0.194	1.818	0.025	0.433	2	1
0.4	1.395	4.739	0.043	0.587	3	1
0.5	1.180	3.941	0.055	0.851	3	1
0.6	3.298	6.383	0.063	1.310	4	1
0.7	2.297	6.230	0.069	2.176	4	1
0.8	0.769	6.609	2256.284	35.273	3	11
0.9	0.259	11.736	114696.807	58.281	2	15
1	5.867	2.706	319065.732	23.571	5	16

γ Degree of risk aversion; y_t^h Threshold level of first period income where investment in health becomes positive; y_t^s Threshold level of first period income when investment in schooling becomes positive; U_t^h Expected lifetime utility when health switches to strictly positive; U_t^s Expected lifetime utility when schooling switches to strictly positive; Gen. t^h generation when investment in health switches to strictly positive; Gen. t^s Generation when investment in schooling switches to strictly positive.

Figure 1.Schooling investment decreasing in risk aversion

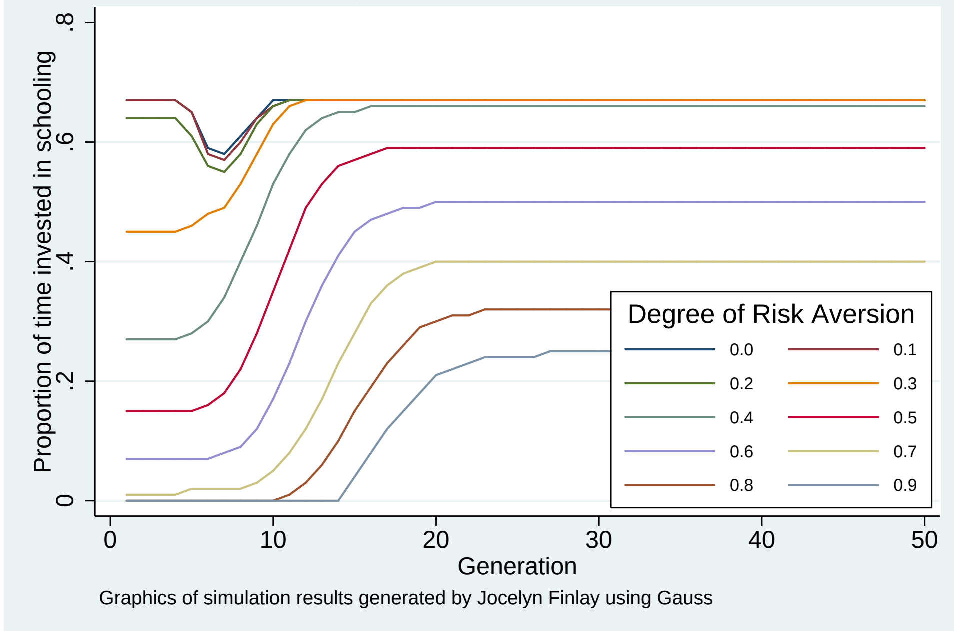

Figure 2. Health investment decreasing in risk aversion