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# **A Theory of Retirement**

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## **Abstract**

We construct a life-cycle model in which retirement occurs at the end of life as a result of declining health. We show that improvements in life expectancy, coupled with a delay in the onset of disability, increases both the optimal consumption level *and* the proportion of life spent in leisure. The retirement age increases proportionally *less* than the increase in life expectancy.

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## 1. Introduction

Improvements in life expectancy and living standards over the last 150 years reflect a dramatic increase in human welfare. For the world as a whole, life expectancy at birth rose from around 30 years in 1900 to 65 years by 2000 (and is projected to rise to 81 by the end of this century; Lee 2003). These improvements have not only resulted in a large direct gain in welfare (Nordhaus 2003, Becker, Philipson, and Soares 2005) but have also had a profound influence on life-cycle behavior through changes in the time horizons of individuals (Hamermesh 1985).

In this paper we extend the Blanchard-Yaari-Weil model of consumption with finite lives and perfect annuity markets to endogenously determine both retirement and the time path of consumption. We use the model to show how optimal retirement and savings decisions respond to changes in wage levels and life expectancy. Standard concavity conditions usually imply the smoothing of leisure over time. The lumping of leisure at the end of life requires some force that deters working when old. We explain retirement as the result of rising ill health with age, which increases the disutility of labor and reduces the productivity of working.

Although health declines with age, rising life spans have been accompanied by improving age-specific health status; that is, our lives are both longer and healthier. The ‘compression of morbidity’ hypothesis (Fries 1980), maintains that the average age at first infirmity, disability, or other morbidity is postponed to such an extent that the period of ill health at the end of life is compressed. Increases in life expectancy in the United States over the last two centuries have indeed been associated with reductions in the age-specific incidence of disease, disability, and morbidity (Costa 2002; Fogel 1994, 1997). This trend is continuing (Crimmins, Saito, and Ingegneri 1997; Freedman, Martin and Schoeni 2002). We model the compression of morbidity by assuming that the age of

onset of disability rises proportionately, or more than proportionately, with life expectancy.

We examine the optimal choices of agents in a model with complete markets. An increase in life span has an "income" effect in that it enlarges the budget set. It also generates substitution effects by changing the rewards to working and saving. Our model shows that, under standard assumptions on preferences, the income effect tends to dominate, producing both increased consumption and leisure when life expectancy increases. Indeed the wealth effect of an increase in life expectancy will reduce the proportion of life span devoted to work, though it is unlikely to reduce the absolute length of the working life. The level of wages also has income and substitution effects. In our model increases in the level of wages will reduce the length of the working life if the worker's inter-temporal elasticity of substitution with respect to consumption is less than one (corresponding to a coefficient of relative risk aversion greater than one).

We use these results to argue that the long-term decline in the age of retirement in industrial countries over the last 150 years (i.e., that the labor force participation rate of men in the United States aged 65 and older declined from around 80 percent in 1850 to less than 20 percent by 2000 (Costa 1998b)) has been due to the increasing *level* of wages, which coincides with Costa's (1998a) analysis.

In addition to offering a positive theory for observed behavior, our approach offers a method through which to estimate the welfare effects of proposed changes to social security systems. Assuming that social security systems desire to promote the optimal retirement and consumption outcomes rational agents would make with complete markets, the results of our model can serve as a benchmark in the design of public pension systems.

A natural approach to dealing with solvency problems that arise in a social security system when life expectancy increases is to increase contribution rates and to

reduce benefit rates. By contrast, our model implies that the optimal response should be to reduce contribution rates and increase benefit rates, maintaining solvency exclusively by increasing the age of retirement, though this age can rise less than proportionately with life expectancy. This response can maintain solvency because rising wages over time and compound interest on accumulated savings mean that longer working lives tend to create more than proportionately higher wealth at retirement. We show that under standard assumptions on interest rates and wage growth, and given projected changes in age specific survival curves, such a plan is feasible.

Several papers have tackled the issue of how changes in longevity affect retirement and consumption decisions. The approach most similar to ours is that of Chang (1991) who argues that in competitive markets a rise in life expectancy reduces consumption and increases savings rates. The major difference between Chang's model and ours is his assumption that the forces that give rise to retirement at each age (low wages and a high disutility of labor) remain unchanged when life expectancy rises. By contrast, we assume that healthy life expectancy also rises, making a longer working life more palatable. Chang (1991) and Kalemli-Ozcan and Weil (2002) also emphasize that with no annuity markets a reduction in mortality rates increases the effective return to saving and encourages saving and early retirement when life expectancy rises. Here we emphasize that the wealth effect, which leads to increased consumption (reduced saving) and more leisure, tends to dominate when markets are complete.

We present our model in section 2, and in section 3 we show how the dynamic programming problem facing agents generates a set of equations (derived from the first-order conditions) that determine the optimal retirement age and consumption profile. In section 4 we use the model to investigate the effect of increases in healthy life span and wages on savings and retirement behavior and present our results. Section 5 discusses

the implications of our results for social security systems and section 6 makes some concluding remarks.

## 2. The Model

This life-cycle theory of saving and retirement is not a complete model of saving and retirement behavior. There are several influences that it does not incorporate. For example, people may engage in precautionary saving to guard against income shocks, and they may also save to provide bequests (Skinner 1985). In addition, whereas in many developing countries the elderly are supported to a large extent by intra-family transfers, in many industrial countries the elderly derive considerable support from social security systems. These transfer systems clearly affect incentives to save and to retire.

Notwithstanding its limitations, the life-cycle model can provide valuable insights into behavior. In addition, optimal decisions by individuals with complete markets generate an efficient outcome that can be used to guide policy even under market imperfections.

Our formal life cycle model makes a number of simplifying assumptions. We assume that the mortality schedule is exogenous, which ignores the possibility of using consumption and health services to extend longevity (Ehrlich and Chuma 1990) or of a reverse link from labor supply to health status and life expectancy. In the interest of simplicity, we also assume that the life cycle has no period of schooling. Schooling will affect the productivity of workers, their health and longevity, and their utility of leisure (Heckman 1976) complicating a retirement - consumption model.

We begin with mortality. We assume that there is a family of possible survival schedules indexed by a single variable  $\lambda$ . There is a survival schedule  $s(t, \lambda)$  that gives the probability of survival to age  $t$ . The mortality rate at time  $t$  is

$$m(t, \lambda) = - \frac{\frac{ds}{dt}(t, \lambda)}{s(t, \lambda)} \quad (1)$$

and life expectancy is given by<sup>1</sup>

$$z(\lambda) = \int_0^T t m(t, \lambda) s(t, \lambda) dt = \int_0^T s(t, \lambda) dt \quad (2)$$

We assume that there is a biological maximum,  $T$ , to the length of life (Carnes, Olshansky, and Grahn, 2003). Let us assume that as the index  $\lambda$  increases, mortality at each age falls, so that survival to each age rises, as does overall life expectancy,  $z$ , and that this relationship is invertible. Lee and Miller (2001) show using empirical data that in practice the Lee and Carter (1992) method of indexing mortality rates generates a monotonic relationship between the index and the age-specific mortality rates; adopting this method of indexing would therefore imply an invertible mapping between the parameter  $\lambda$  and life expectancy. Assuming a one-to-one mapping from life expectancy to our index of mortality, we can take life expectancy to be a unique identifying index for a mortality schedule. In what follows we therefore use life expectancy at the beginning of working life,  $z$ , as our index of the mortality schedule and write mortality<sup>2</sup> as  $m(t, z)$ .

A major innovative feature of our model is the health schedule  $h(t, z)$ . We assume that health declines with age  $t$ . In addition, we postulate that health at age  $t$  depends on life expectancy,  $z$ . If health is independent of  $z$ , it implies that increases in life expectancy are not associated with general health improvements in the form of reductions in morbidity (sickness), which implies further that population aging is associated with an increasingly large share of unhealthy people.

However, evidence points in the opposite direction. Not only are people living longer, but age specific disability is also falling. Cutler (2001) attributes reductions in

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<sup>1</sup> The probability that your age of death is exactly  $t$  is the probability of surviving to  $t$  times the mortality rate at  $t$ . The second equality comes from substituting in for the mortality rate and integrating by parts using the fact that  $s(0, \lambda) = 1$  and  $s(T, \lambda) = 0$ .

<sup>2</sup> Note that we do not impose the Lee-Carter functional form; we only specify that there is a one-to-one mapping from mortality schedules to life expectancy so that life expectancy identifies a mortality schedule.

disability at older ages to long-term protective effects of reduced disease exposure in childhood, rising levels of education and socio-economic status, improved health-related behavior such as reductions in smoking, improved medical care, and the use of special aids to reduce disability for a given health state. Manton, Gu, and Lamb (2006) estimate that between 1935 and 1982 life expectancy and (disability-free) active life expectancy at age 65 both moved up in the United States, keeping the proportion of disability-free life after 65 relatively steady at around 73%. However, between 1982 and 1999 life expectancy at age 65 rose from 16.9 to 17.7 years while active life expectancy rose from 12.3 to 13.9 years. This implies a rise in the proportion of disability-free life to over 78%, and a reduction in the absolute time spent in disability from 4.6 to 3.8 years. The compression of morbidity, either absolute or relative, appears to hold in most developed countries (Mor 2005). Although time-series evidence in developing countries is scarce, Mathers et al. (2001) show that across countries health-adjusted life expectancy (each life year weighted by a measure of health status) increases as fast as total life expectancy, implying a rising proportion of healthy to total life expectancy as life expectancy increases.

Although the compression of disability, at least in the recent past, in the United States seems evident, and although disability is a cause of retirement (Gordo 2006), most people retire before the onset of severe disability. Despite the fact that they may still be physically capable of working, moderate ill health not amounting to disability may deter their participation in the labor force. The “dynamic equilibrium” hypothesis suggests the compression of time spent in severe disability may be the result of reducing the transition rate from low or moderate ill health to severe disability. This in turn may result in an increase in the prevalence of less-severe disability states (e.g., Graham et al. 2004).

To examine the effect of the compression of morbidity we begin by taking as our benchmark the case where healthy life span increases proportionately with overall life span. We assume that:

$$h(\rho t, \rho z) = h(t, z) \text{ for } \rho > 0 \quad (3)$$

Note that we assume both age and life expectancy are measured from the start of adult life. If a particular level of health is labeled “disabled,” equation (3) implies that the age of onset of this “disabled” health rises proportionately with life expectancy. We examine this case below, and discuss how our results extend to the case where healthy life spans increase faster than life expectancy allowing for the absolute compression of morbidity.

We denote labor supply at time  $t$  by  $\chi_t$ , which is assumed to lie in the closed interval  $[0,1]$ . Utility is assumed to be strongly separable in goods and leisure and is given by

$$u(c(t)) - \chi(t)v(h(t, z)) \quad (4)$$

where  $c(t)$  is consumption at age  $t$  and  $v(h(t, z))$  is the disutility of working given the health state  $h(t, z)$ . We assume that  $u$  is twice differentiable with  $u'(c) > 0, u''(c) < 0$  and that the disutility function  $v$  satisfies  $v'(h) < 0$  so that the disutility of work (and the relative utility of leisure) is higher when health is poorer. Lifetime expected utility is

$$U = \int_0^T e^{-\delta t} s(t, z) [u(c(t)) - \chi(t)v(h(t, z))] dt \quad (5)$$

where  $\delta$  is the subjective rate of time preference, and  $T$  is the biological maximum life span. The wage earned by a worker with health  $h$  is given by

$$w(t, h) = w_1(t)w_2(h(t, z)) \quad (6)$$

The term  $w_1(t)$  captures the change in wages over time due to exogenous forces, for example, technical progress, and  $w_2(h(t, z))$  captures the fact that worker productivity

depends on health, which is a function of age relative to life expectancy. We assume  $w_1'(t) \geq 0$  and  $w_2'(h) \geq 0$  so that wages are increasing over time due to exogenous forces and are increasing in health. The multiplicative functional form implies that the health effect on log wages is additive, which is consistent with a Mincer wage equation that includes health as a form of human capital as in Schultz (1999). Chang (1991) and Kalemli-Ozcan and Weil (2002) have exogenous age-wage schedules that do not change as life expectancy varies; we allow for an effect of life expectancy on wages through improved health.

Wealth,  $W$ , evolves according to

$$\frac{dW}{dt} = \chi(t)w_1(t)w_2(h(t, z)) + (m(t, z) + r)W(t) - c(t) \quad (7)$$

If the agent works at time  $t$ , he or she earns the wage  $w_1(t)w_2(h(t, z))$ , and this is added to wealth. Consumption,  $c(t)$ , reduces wealth, while the stock of wealth earns a return,  $m(t, z) + r$ . We assume that health may affect labor productivity and wages. We assume that wealth can be transferred from one period to another by saving or borrowing from the financial sector. This competitive financial sector can borrow or lend freely at the interest rate  $r$ . Agents are paid an effective interest rate  $m(t, z) + r$  on their savings, which is larger than  $r$ , to compensate them for the fact that they may die before withdrawing their savings. Similarly, agents who borrow pay the rate  $m(t, z) + r$  to compensate the bank for the fact that they may die before repaying their borrowings. This is equivalent to treating all savings as being in the form of annuity purchases, while assuming that all borrowing has to be accompanied by an actuarially fair life insurance contract for the amount of the loan.

Provided that a continuum of agents exists, the financial sector can avoid all risk by aggregating over individuals thereby earning zero profits. The transfer of wealth from

those who die to the financial sector exactly compensates deposit-taking institutions for the fact that they pay an interest rate  $m(t, z) + r$  on deposits that exceed the risk-free rate  $r$ , and rules out the need to consider unintended bequests. The budget constraint is:

$$\int_0^T e^{-rt} s(t, z) c(t) dt \leq \int_0^T e^{-rt} s(t, z) \chi(t) w_1(t) w_2(h(t, z)) dt \quad (8)$$

The control variables for the agent's optimization problem are  $c$  and  $\chi$ . Agents must decide when to work and what their consumption stream should be.<sup>3</sup>

We assume the boundary conditions:

$$\begin{aligned} w_1(0)w_2(h(0, z))u'(0) &> v(h(0, z)) \\ w_1(T)w_2(h(T, z))u'(0) &< v(h(T, z)) \end{aligned} \quad (9)$$

The first boundary condition implies that the disutility of working at age zero is sufficiently low to ensure that the agent always prefers to work at the start of life, rather than live with no consumption. The second boundary condition implies that the disutility of working at the maximum possible age is sufficiently high that the agent always prefers to be retired.

It what follows we will assume that the rate of interest,  $r$ , is positive and equals the rate of time preference,  $\delta$ . Our theory is developed for the case where the growth rate of the

exogenous component of wages is moderate. If wage growth is very rapid a retired worker may reenter the labor market to take advantage of the high wages when old. To rule this out we assume that for every life expectancy, the rate of growth of the disutility of labor with age exceeds the exogenous component in the rate of growth of wages at each point in time:

$$\frac{\dot{w}_1}{w_1} < \frac{\dot{v}}{v} \quad (10)$$

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<sup>3</sup> Adding the direct utility of health as an additive term to the utility function does not affect decision making in any way.

We show that this condition will imply that retirement occurs only once and is never reversed. In this case we can label the retirement age as  $R$ .

### 3. Optimal Retirement and Consumption Decisions

The Hamiltonian for this problem is

$$H = e^{-\delta t} s(t, z) [u(c(t)) - \chi(t)v(h(t, z))] + \phi(t) [\chi(t)w_1(t)w_2(h(t, z)) + (m(t, z) + r)W(t) - c(t)] \quad (11)$$

The first-order conditions for a maximum in  $c$  and  $\chi$  are:

$$\dot{\phi} = -\frac{\partial H}{\partial W} = -\phi(t)(r + m(t, z)), \quad (12)$$

$$\frac{\partial H}{\partial c} = e^{-\delta t} s(t, z) u'(c(t)) - \phi(t) = 0, \text{ and} \quad (13)$$

$$\begin{aligned} \frac{\partial H}{\partial \chi} &= -e^{-\delta t} s(t, z) v(h(t, z)) + \phi(t) w_1(t) w_2(h(t, z)) \geq 0 \text{ when } \chi(t) = 1 \\ \frac{\partial H}{\partial \chi} &= -e^{-\delta t} s(t, z) v(h(t, z)) + \phi(t) w_1(t) w_2(h(t, z)) = 0 \text{ when } \chi(t) \in (0, 1) \\ \frac{\partial H}{\partial \chi} &= -e^{-\delta t} s(t, z) v(h(t, z)) + \phi(t) w_1(t) w_2(h(t, z)) \leq 0 \text{ when } \chi(t) = 0 \end{aligned} \quad (14)$$

These conditions can be shown to yield the following<sup>4</sup>:

$$\frac{\dot{c}}{c} = (r - \delta) \frac{u'(c)}{-cu''(c)} \quad (15)$$

$$\begin{aligned} u'(c(t))w_1(t)w_2(h(t, z)) &> v(h(t, z)) \Rightarrow \chi(t) = 1 \\ u'(c(t))w_1(t)w_2(h(t, z)) &= v(h(t, z)) \Rightarrow \chi(t) \in [0, 1] \\ u'(c(t))w_1(t)w_2(h(t, z)) &< v(h(t, z)) \Rightarrow \chi(t) = 0 \end{aligned} \quad (16)$$

The first condition is standard and arises because the annuities perfectly insure against financial losses due to death. Given our assumption that  $r = \delta$ , optimal

<sup>4</sup> Equation (15) follows from equation (1). We suppress the fact that consumption depends on  $t$  though this is implicit in equation (15).

consumption is constant over time. The second condition implies that agents work at time  $t$  so long as the utility gain from the consumption purchased by the wage they earn (the marginal utility of consumption times the wage) exceeds the disutility of working.

The concavity of the utility function  $u$  implies that the Hamiltonian is concave in the two control variables. Hence these first-order conditions give a global maximum of the Hamiltonian in the control variables,  $c$  and  $\chi$ . The Hamiltonian is jointly concave<sup>5</sup> in the two control variables and the state variable, wealth, and hence satisfies the Mangasarian condition for our set of equations to give a maximum for the dynamic programming problem (see Sydsaeter, Strøm and Berck 2000).

#### 4. Results

**Proposition 1** (a) It is optimal for an agent to begin life working, retire at age  $R$ , and not to work thereafter. (b) The retirement age and consumption stream that maximize expected lifetime utility are unique.

Proof: see appendix 1.

We first prove that any optimal plan has a single and irreversible retirement age. We then prove that this optimal plan is unique. Note that retirement emerges endogenously from our model. In our framework a lack of health effects on the disutility of labor and wages, combined with exogenous wage growth, would tend to produce leisure in youth when wages are low, and work when old and wages are high; the health effects are essential for our model to produce retirement.

At the beginning of life the disutility of labor is low and the benefits of working in terms of additional consumption spread over the life cycle are large. As life proceeds, rising disability eventually outweighs the benefits of working, and retirement occurs.

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<sup>5</sup> The Hessian of second derivatives is negative semi-definite everywhere.

Given proposition 1 we can rewrite the budget constraint as:

$$\int_0^T e^{-rt} s(t, z) c(t) dt \leq \int_0^R e^{-rt} s(t, z) w_1(t) w_2(h(t, z)) dt \quad (17)$$

In what follows we think of the choice variables as the consumption level and the retirement age, with these chosen subject to equation (17). Consumption is steady over the life cycle. Rising wages for young workers can lead to dis-saving when young before saving in middle age in preparation for retirement when old, as occurs in the examples shown in appendix 1.

Consider what happens if life expectancy increases. Lee and Goldstein (2003) argue that a natural benchmark for responding to an increase in life expectancy is to adjust the timing of all life course choices proportionately. Bloom, Canning, and Graham (2003) show that in the case of a certain life span, the optimal response to an increase in life span and health is to increase working life proportionately keeping consumption unchanged, provided that interest rates, time preference rates, and exogenous wage growth are all zero. In our framework, however, it is not clear that such a proportional response is even feasible. Consider an initial life expectancy  $z_0$  and let the optimal retirement age be  $R_0^*$  and the optimal consumption level be  $c_0^*$ . Let the optimal retirement age at the longer life expectancy,  $z_1$ , be  $R_1^*$  and the optimal consumption level be  $c_1^*$ . We require that the old consumption stream is affordable at the new survival schedule provided retirement is postponed in line with life expectancy.<sup>6</sup> We call this the *proportionality condition*:

$$\int_0^T e^{-rt} s(t, z_1) c_0^* dt \leq \int_0^{R_0^* z_1 / z_0} e^{-rt} s(t, z_1) w_1(t) w_2(h(t, z_1)) dt \quad \text{for } z_1 > z_0 \quad (18)$$

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<sup>6</sup> Note that by proposition 1 we can summarize labor supply by a retirement age. We integrate labor income over time up to retirement to get the value of lifetime earnings.

This proportionality condition may not be satisfied if the extension in life expectancy is due to a rise in the survival schedule concentrated in the period immediately after retirement. However, if the gain in life span comes from a more general reduction in mortality rates that generates survival increases during the working life (which add to labor income) or near the end of life (where consumption is heavily discounted), the additional income generated by proportionately later retirement will be sufficient to keep consumption unchanged.

In practice, this condition is satisfied in historical data and future projections for the United States. In Appendix 2 we examine cohort survival schedules for the United States (the survival curve faced by the cohort) for different birth years. Using long run averages for the real interest rate and wage growth, we show that moving from the survival schedule of those born in 1900 to those born in 1925, 1950, 1975, or 2000 (based on historical cohort survival rates and projected rates) with an increase in the retirement age proportional to the increase in life expectancy, would in each case allow consumption to increase. This means that it has been feasible to keep the working life proportional to life expectancy with consumption unchanged.

We begin by giving results for the relative compression of morbidity as set out in equation (3). We will later extend this result to the case in which compression of morbidity is more than proportional.

**Proposition 2:** If there is proportional compression of morbidity, and changes in the survival schedule satisfy the proportionality condition, the optimal response to an increase in life expectancy requires consumption to stay the same or increase.

Proof: see Appendix 3.

The intuition for this result is that it is possible for the agent to increase working life proportionately but keep consumption unchanged. In general the agent will do better than this. If both leisure and consumption are normal goods the agent will take some of the additional welfare in the form of consumption and some in the form of increased leisure.

**Proposition 3:** If changes in the survival schedule satisfy the proportionality condition, the optimal response to an increase in life expectancy requires the retirement age to rise less than proportionately to life expectancy if and only if

$$\frac{\dot{w}_1}{w_1} < \frac{1}{R} \left[ \frac{1}{\theta(c(R, z))} \right] \left( \frac{R}{c} \frac{\partial(c(R, z))}{\partial R} + \frac{z}{c} \frac{\partial(c(R, z))}{\partial z} \right) \quad (19)$$

where  $\theta(c(R, z))$  is the inter-temporal elasticity of substitution at the consumption level  $c(R, z)$ .

Proof: see appendix 4.

The proportionality condition implies that an increase in life expectancy, coupled with a proportional increase in the retirement age, is possible without consumption falling. Hence

$$\left( \frac{R}{c} \frac{\partial(c(R, z))}{\partial R} + \frac{z}{c} \frac{\partial(c(R, z))}{\partial z} \right) \geq 0$$

If there is no growth in the exogenous component of wages, then the proportion of life expectancy spent working must stay the same or fall. This is the wealth effect of a longer healthy life span, leading to a rise in both consumption and the relative proportion of time spent in leisure. Although rising consumption levels deter labor supply, this effect can be offset by the substitution effect of a rapid increase in the wage rate.

Condition (19) guards against the possibility that wages are rising so rapidly over time that the substitution effect dominates the wealth effect and workers increase the proportion of life spent working to take advantage of the higher wages later in life.

We now turn to the issue of how retirement and consumption are affected by a change in the exogenous component of the wage schedule. We consider what happens when the levels of wages rises; we think of the wage schedule moving up proportionately at every point in time. We find that what happens depends crucially on the inter-temporal elasticity of substitution, which in our model is the inverse of the coefficient of relative risk aversion because of the assumption of time-separable utility.

**Proposition 4:** When the level of wages rises the optimal response of the agent is to

(i) keep retirement unchanged and increase consumption proportionally with wages if the utility function has a local inter-temporal elasticity of substitution of unity.

(ii) increase the age of retirement and increase consumption more than proportionally with wages if the utility function has a local inter-temporal elasticity of substitution greater than unity.

(iii) decrease the age of retirement and increase consumption less than proportionally with wages if the utility function has a local inter-temporal elasticity of substitution less than unity.

Proof: see Appendix 5.

It is well known that the impact of wages on consumption and the demand for leisure has both income and substitution effects. The pivotal case occurs where there is relative risk aversion that is locally unity. We state this proposition to show that the result applies to our model. The long-term decline in the retirement age, and increase in

retirement savings, is consistent with an inter-temporal elasticity of substitution less than unity (which in our model implies a coefficient of relative risk aversion that exceeds one).

We now turn to the case where we have health improvements that are greater than proportional to the rise in life expectancy:

$$h(\rho t, \rho z) > h(t, z) \text{ for } \rho > 1 \quad (20)$$

This allows for age specific health status to rise more than proportionately with life expectancy and is compatible with an absolute compression of morbidity. Proposition 1 clearly continues to hold because it investigates the optimal retirement path for a fixed life expectancy. Similarly proposition 4 reflects the effect of changing wage levels with a fixed life expectancy and so does not depend on how health changes with life expectancy.

Proposition 2, which argues that consumption must rise when life expectancy and health rise proportionately, continues to hold. Condition (3) is used at only one point in the proof in a chain of inequalities and this chain continues to hold if we employ condition (20) instead. The intuition is that with condition (20) the wealth effect gets larger, and wages at each age rise with better health allowing higher consumption. Only proposition 4 changes if we adopt condition (20) instead of condition (3). The higher wages and lifetime consumption will again promote earlier retirement relative to life expectancy, but the better health at older ages increases productivity and wages, while reducing the disutility of labour. These health effects, coupled with rising exogenous wages, can create a substitution effect promoting work that may result in working life rising faster than proportionately than life expectancy.

We focus on the effects of changes in life expectancy and the level of wages on retirement and saving decisions. Bloom, Canning, and Moore (2004) also investigate the

effects of changes in the interest rate, rate of time preference, and the rate of wage growth under more restrictive functional forms for utility and mortality.

## **5. Implications for Social Security**

Our results give the optimal retirement and savings behavior of fully rational agents under complete capital markets. In practice, imperfect capital markets (particularly annuity markets), lack of foresight about the need to save for retirement, and time inconsistency in preferences will distort private decisions, generating a role for social security systems (Hubbard and Judd 1987; Feldstein 1985; Laibson 1998; Laibson et al. 1998). In these circumstances our model will fail to predict behavior, but it can nonetheless serve as a benchmark for the design of social security systems whose objective is to mimic the optimal plan.

When life expectancy rises, existing social security arrangements often become financially unsustainable. A common argument is that the resolution of this problem requires some combination of increased contributions, reduced benefits, and extended working life, with a division of the burden of adjustment across all three margins. The problem with this "balanced" approach is that it increases leisure, because the retirement age does not go up as fast as life expectancy, while reducing consumption, both when working (to pay the higher contributions) and in retirement (due to lower benefits).

Our analysis suggests that an increase in life spans enlarges the available budget set and the optimal response to this wealth effect is an increase in both leisure and consumption. Social security can mimic this by reducing contribution rates, increasing benefit rates, and raising the retirement age. Financial sustainability should be maintained entirely through a longer working life, though the effects of compound

interest and rising wages over time mean that the working-life extension can be less than proportional to the increase in life expectancy.

This assumes that the goal of the social security system is to mimic the individually optimal plan. The system may also have a goal of redistribution of income within a cohort or across cohorts (Kotlikoff, Spivak, and Summers 1982). With redistribution, the social planner may face a different budget constraint than the individual. Unless the transfers are lump sum, the implicit tax and subsidy rates will distort behavior.

Mandatory retirement may prevent individuals from responding optimally to longer life spans through longer working lives. Although few social security systems have mandatory retirement, a substantial body of evidence indicates that retirement in industrial countries clusters around specific ages that depend on retirement incentives inherent in the national social security system (Gruber and Wise 1998). Mandatory retirement, or incentives that amount to the same thing, are not required in social security systems. It is possible to sever the link between social security pensions and stopping work, or to allow a longer working life with an actuarially fair benefit adjustment when retirement does occur. The mandatory retirement element of a social security system has a welfare cost. Adding a retirement constraint to the optimization problem given by equation (11) generates a shadow price for this constraint. Suppose the mandatory retirement age is  $R^*$ , which is less than the retirement age the agent would optimally choose. In current consumption units, the shadow price of the constraint (and the agent's willingness to pay to relax the constraint is):

$$w_1(R^*)w_2(h(R^*, z)) - \frac{v(h(R^*, z))}{u'(c(R^*, z))} \quad (21)$$

As life expectancy and health improve,  $w_2(h(R^*, z))$  rises and  $v(h(R^*, z))$  falls, increasing the welfare cost of the retirement constraint. If the rise in life expectancy is generated by gains in survival concentrated in the working years, then consumption may rise; however, gains in survival after retirement usually will force consumption to fall, raising the marginal utility of consumption  $u'(c(R^*, z))$  and further increasing the shadow price on mandatory retirement.

If the institutional retirement age does not rise with life expectancy, individuals may be forced to save more for their longer retirement period. Bloom, Canning, Mansfield, and Moore (2007) examine empirically how changes in life expectancy affect savings behavior under different social security arrangements.

## 6. Conclusion

Our model demonstrates two major long-term influences on the optimal age at retirement and on savings behavior. First, at higher levels of lifetime wages, the desire for increased leisure leads to early retirement funded through a higher savings rate. Second, longer life spans and healthier lives lead to higher consumption and lower savings rates that can be consistent with a less-than-proportional increase in working life.

We make a number of simplifying assumptions to derive these results. Our model assumes that the instantaneous utility function is additively separable in leisure and consumption, that the lifetime utility function is strongly inter-temporally separable, and that the interest rate equals the rate of time preference. We also treat health as changing deterministically with age. Future work could allow for the effects of schooling, random health shocks, raising issues of precautionary saving and insurance of health care costs.

## Appendix 1

**Proposition 1:** (a) Given our assumptions the optimal plan for the agent is to begin life working, retire at an age  $R$ , and not to work thereafter.

At point  $R$  the agent is indifferent between working and retiring and the equality condition of equation (16) holds. Given equation (15) and the fact that  $r = \delta$ , optimal consumption is constant over the life span. Now consider the function

$$\psi(R) = u'(c)w_1(R)w_2(h(R, z)) - v(h(R, z)) \quad (22)$$

Clearly for  $R$  to be the optimal retirement and to satisfy equation (16) we require  $\psi(R) = 0$ . We wish to show that only one  $R$  can satisfy the condition that  $\psi(R) = 0$  so the retirement age is unique and retirement is permanent. We do this by showing that  $\psi'(R) < 0$  whenever  $\psi(R) = 0$ . Thus every point where  $\psi(R) = 0$  is a transition from working to retirement and re-entry to the labor market once retired is impossible.

We proceed by contradiction. Assume  $\psi'(R) \geq 0$ . We have

Hence at any  $R$  with  $\psi(R) = 0$  we have

$$\begin{aligned} & \frac{d \log(w_1(R))}{dR} + \frac{d \log(w_2(h(R, z)))}{dh} \frac{dh(R, z)}{dR} \\ & \geq \frac{d \log(v(h(R, z)))}{dR} \end{aligned} \quad (24)$$

Because  $h(R, z)$  is decreasing in  $R$  while  $w_2(h)$  is increasing in  $h$ , the second term on the left hand side of equation (24) is negative. Because the growth of the exogenous component of wages and the disutility of labor with respect to the retirement age is the same as with respect to time, equation (24) implies

$$\frac{\dot{w}_1}{w_1} \geq \frac{\dot{v}}{v} \quad (25)$$

which contradicts the assumption in equation (10).

Hence  $\psi'(R) < 0$  at every age  $R$  with  $\psi(R) = 0$ . It follows that ages that satisfy the first-order condition (where  $\psi(R) = 0$ ) are isolated. Now take two adjacent points  $R_1 < R_2$  that satisfy  $\psi(R) = 0$ . By the fact that the derivative of  $\psi$  is negative at both points there exists  $\varepsilon$  small such that  $\psi(R_1 + \varepsilon) < 0$  and  $\psi(R_2 - \varepsilon) > 0$ . Hence by the intermediate value theorem there exists  $R_3$  between  $R_1$  and  $R_2$  with  $\psi(R_3) = 0$ . This contradicts  $R_1$  and  $R_2$  being adjacent. It follows that there can be at most one age  $R$  satisfying  $\psi(R) = 0$  and that the worker works up to this age and does not work afterwards. We call  $R$  the retirement age.

Now consider the boundary conditions. These imply the worker does not work at  $T$ . Now suppose the worker never works. Then consumption is always zero and the other boundary condition implies that the worker prefers to work at time zero, a contradiction.

Therefore the worker starts life working, and retirement occurs once and is permanent.

(b) The retirement age and consumption stream that maximize expected lifetime utility are unique. Note that in the proof of part (a) we used the fact that in an optimal plan consumption is flat over time. We now, however, compare two optimal plans, one with a longer life expectancy and one with a shorter one. By part (a) each optimal plan has a unique retirement age. From the budget constraint it is clear that a later retirement age will generate higher lifetime consumption. Again we consider the function  $\psi(R)$  over different possible retirement ages

$$\psi(R) = u'(c(R))w_1(R)w_2(h(R, z)) - v(h(R, z)) \quad (26)$$

We require  $\psi(R) = 0$  at an optimal retirement age. Once again, assuming  $\psi'(R) \geq 0$ , we have

$$\begin{aligned} \psi'(R) = & u''(c(R)) \frac{dc(R)}{dR} w_1(R) w_2(h(R, z)) + u'(c(R)) \frac{dw_1(R)}{dR} w_2(h(R, z)) \\ & + u'(c(R)) w_1(R) \frac{dw_2(h(R, z))}{dh} \frac{dh(R, z)}{dR} - \frac{dv(h(R, z))}{dh} \frac{dh(R, z)}{dR} \geq 0 \end{aligned} \quad (27)$$

Now, however, we have  $\frac{dc(R)}{dR} > 0$  because we are comparing retirement over different optimal paths rather multiple retirement ages within a single optimal path. Using the fact that  $\psi(R) = 0$  at each retirement age we now have

$$\left[ \frac{-1}{\theta(c(R))} \right] \frac{d \log(c(R))}{dR} + \frac{d \log(w_2(h(R, z)))}{dh} \frac{dh(R, z)}{dR} + \frac{\dot{w}_1}{w_1} \geq \frac{\dot{v}}{v} \quad (28)$$

where  $\theta(c(R))$  is the local inter-temporal elasticity of substitution at retirement, which is positive because the utility function is concave. Because  $c(R)$  is increasing and  $w_2(R, z)$  is decreasing in  $R$ , the first two terms on the left hand side of equation (28) are negative. Hence equation (25) must again hold, which contradicts our assumptions. It follows that in this case we also have  $\psi'(R) < 0$  at each optimal retirement age. By a similar argument to that in part (a) we have that  $\psi(R) = 0$  only once and that there is only one optimal plan for the agent.

## Appendix 2

### Proportionality Condition:

To investigate this we look at the empirical survival schedules for the United States. Let us assume that there is no health effect on wages; the health effect will increase wage

income when life expectancy rises making the proportionality condition easier to satisfy. We assume real wages grow at a constant rate  $\sigma$ . Then the budget constraint gives the initial ratio of consumption to wages at the original survival schedule

$$\frac{c_0^*}{w_0} = \frac{\int_0^{R_0^*} e^{(\sigma-r)t} s(t, z_0) dt}{\int_0^{\infty} e^{-rt} s(t, z_0) dt} \quad (29)$$

This implies that the proportionality condition on the initial consumption-wage ratio is:

$$\frac{\int_0^{R_0^* z_1 / z_0} e^{(\sigma-r)t} s(t, z_1) dt}{\int_0^{\infty} e^{-rt} s(t, z_1) dt} \geq \frac{\int_0^{R_0^*} e^{(\sigma-r)t} s(t, z_0) dt}{\int_0^{\infty} e^{-rt} s(t, z_0) dt} \quad (30)$$

This can be easily checked given two survival schedules provided that we have data for the real interest rate, the rate of real wage growth, and the initial retirement age. In its fiscal forecasts for Social Security in the US, the Congressional Budget Office (2004) uses historical averages for the real interest rate of 3.30% per annum and of the rate of real wage growth of 1.27% per annum. We use the same figures. Gendell (2001) shows that between 1950 and 2000 the mean retirement age for men fell from just under 69 years to just under 63 years of age. Costa (1998a) shows that labor force participation rates for men over 65 before 1900 were around 80% but had fallen to around 50% by 1950, suggesting a high mean age of retirement before 1900. We take a range of possible figures for the retirement age in 1900 (80, 70, and 60 years of age) to allow for variations across individuals.

We use data on survival rates for cohorts born in 1900, 1925, 1950, 1975, and 2000 based on mortality data and projections from the Office of the Actuary of the Social Security Administration. These data are available from the Berkeley Mortality

Database.<sup>7</sup> Figure 1 shows the survival curves for each of these cohorts. The life expectancy of men in these cohorts rises from 51.5 years in 1990 to 78.2 by the year 2000.<sup>8</sup>

For each fixed retirement age we ask if a proportional increase in the age of retirement (proportional to life expectancy) would allow the same consumption stream as before if given a later survival schedule. Table 1 shows the results from such an exercise starting in 1900. We assume that adult life starts at age 16. The first row in the table gives the years we use for comparison. The second row shows life expectancy at age 16 in each year. The third row gives the ratio of life expectancy at age 16 in each year relative to the figure for 1900. This ratio is how much the retirement age has to be increased if the ratio of working life to adult life is to be kept constant.

We now turn to our calculation of the initial consumption-wage ratios. The third column of the fourth row gives the initial consumption to wage ratio (at age 16) for someone born in 1900 who starts working at 16 and retires at age 60. This is calculated using the right-hand side of equation (29) based on the survival schedule for someone born in 1900, with a real interest rate of 3.3% and real wage growth of 1.27% per year. Note that the initial consumption-wage ratio is 1.14 so that the worker (initially) consumes 14% more than his wage income. This is a common feature of optimal consumption when real wages are rising. The higher expected wages later in working life allow high initial consumption. The young worker borrows initially to allow high consumption but consumes less than his wage income as an older worker to repay this borrowing and save for retirement.

The next column of row 4 shows the initial consumption-wage ratio the worker could afford if he had the survival schedule of someone born in 1925, but also increased

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<sup>7</sup> The database is available online at <http://www.demog.berkeley.edu/~bmd/>

<sup>8</sup> These cohort life expectancies are larger than the corresponding period life expectancies because the cohort figures take into account falling mortality rates over time as the cohort ages, while the more common period life expectancies are based on current cross sectional mortality rates.

his retirement age by 11% to compensate for the longer adult life expectancy. With this longer working life and total life span, the initial consumption-wage ratio can rise to 1.19 indicating that the proportionality condition is satisfied. We repeat the analysis comparing the cohort born in 1900 with the cohorts born in 1950, 1975, and 2000 and in each case find the proportionality condition is satisfied. We also find that the proportionality condition holds in pair-wise comparisons (not reported) of earlier versus later cohorts based on the survival schedules for cohorts born in 1925, 1950, 1975, and 2000.

### Appendix 3

**Proposition 2:** If changes in the survival schedule satisfy the proportionality condition, the optimal response to an increase in life expectancy requires consumption to stay the same or increase.

Proof. Let  $R_0^*, c_0^*$  be the optimal retirement-consumption plan at life expectancy  $z_0^*$  and let  $R_1^*, c_1^*$  be the optimal retirement-consumption plan at life expectancy  $z_1^*$ . We derive a proof by contradiction.

Let us assume  $c_1^* < c_0^*$ . Let  $z_1 = \theta z_0$  where  $\theta > 1$ . We must have  $R_1^* < R_0^* z_1 / z_0 = \rho R_0^*$ , giving more leisure, else the optimal plan cannot be as good as the proportional plan  $(c_0^*, \rho R_0^*)$ , which is feasible. At the optimal retirement ages from the first-order condition (16) we have

$$u'(c_0^*)w_1(R_0^*) = \frac{v(h(R_0^*, z_0))}{w_2(h(R_0^*, z_0))}$$

$$u'(c_1^*)w_1(R_1^*) = \frac{v(h(R_1^*, z_1))}{w_2(h(R_1^*, z_1))}$$

(a) We first take the case where  $R_1^* \geq R_0^*$ . Because  $R_1^* < \rho R_0^*$ ,  $v$  is strictly increasing and  $w_2$  is strictly decreasing it follows that

$$\frac{v(h(R_1^*, z_1))}{w_2(h(R_1^*, z_1))} < \frac{v(h(\rho R_0^*, \rho z_0))}{w_2(h(\rho R_0^*, \rho z_0))} = \frac{v(h(R_0^*, z_0))}{w_2(h(R_0^*, z_0))}$$

Hence  $u'(c_1^*)w_1(R_1^*) < u'(c_0^*)w_1(R_0^*)$ . Further, because  $R_1^* \geq R_0^*$  we have  $w_1(R_1^*) \geq w_1(R_0^*)$  because  $w_1$  is increasing. Hence  $u'(c_1^*) < u'(c_0^*)$  and so  $c_1^* > c_0^*$ , a contradiction.

(b) Now we examine the case  $R_1^* < R_0^*$ . Again assume  $c_1^* < c_0^*$ .

$$u'(c_1^*)w_1(R_0^*) > u'(c_0^*)w_1(R_0^*) = \frac{v(h(R_0^*, z_0))}{w_2(h(R_0^*, z_0))} > \frac{v(h(R_0^*, z_1))}{w_2(h(R_0^*, z_1))}$$

By our optimality conditions (16) this implies that the agent with life expectancy  $z_1$  works at time  $R_0^*$ . This is a contradiction with  $R_1^* < R_0^*$  and proposition 1 which ensures non-re-entry into the labor market after retirement age is unique. Hence if  $R_1^* < R_0^*$  we must have  $c_1^* \geq c_0^*$ .

#### Appendix 4

**Proposition 3:** If changes in the survival schedule satisfy the proportionality condition, the optimal response to an increase in life expectancy requires the retirement age to rise less than proportionately than life expectancy if and only if

$$\frac{\dot{w}_1}{w_1} < \frac{1}{R} \left[ \frac{1}{\theta(c(R))} \right] \left( \frac{R}{c} \frac{\partial(c(R, z))}{\partial R} + \frac{z}{c} \frac{\partial(c(R, z))}{\partial z} \right)$$

Proof: consider the function  $\psi(R, z)$  where

$$\psi(R, z) = u'(c(R, z))w_1(R)w_2(h(R, z)) - v(h(R, z)) \quad (31)$$

Note that in the proof of proposition 1 we thought of  $\psi$  as a function of  $R$  alone because  $z$  was fixed; we do not let both vary. We require  $\psi(R, z) = 0$  at an optimal retirement age. We begin by showing  $\frac{\partial \psi}{\partial R} < 0$  when  $\psi(R, z) = 0$ ; this is useful later.

$$\begin{aligned} \frac{\partial \psi}{\partial R} = & u''(c(R, z)) \frac{dc(R, z)}{dR} w_1(R) w_2(h(R, z)) + u'(c(R, z)) \frac{dw_1(R)}{dR} w_2(h(R, z)) \\ & + u'(c(R, z)) w_1(R) \frac{dw_2(h(R, z))}{dh} \frac{\partial h(R, z)}{\partial R} - \frac{dv(h(R, z))}{dh} \frac{\partial h(R, z)}{\partial R} \end{aligned} \quad (32)$$

Now using the fact that  $\psi(R, z) = 0$  at each retirement age we have

$$\frac{\partial \psi}{\partial R} = \left[ \frac{-1}{\theta(c(R, z))} \right] \frac{d \log(c(R, z))}{dR} + \frac{d \log(w_2(h(R, z)))}{dh} \frac{\partial h(R, z)}{\partial R} + \frac{\dot{w}_1}{w_1} - \dot{v} \quad (33)$$

where  $\theta(c(R, z))$  is the local inter-temporal elasticity of substitution at retirement, which is positive because the utility function is concave. Later retirement allows extra consumption (via the budget constraint) so  $c(R, z)$  is increasing in  $R$ . Wages  $w_2(h(R, z))$  are increasing in health  $h$ , but health is declining in the retirement age  $R$ . It follows that the first two terms in the left hand side of equation (28) are negative. Using our assumption that wage growth is less than the growth rate of the disutility of working we have  $\frac{\partial \psi}{\partial R} < 0$ .

At the optimal retirement age we have  $\psi(R, z) = 0$ , which defines the optimal retirement age implicitly for each life expectancy  $z$ . By the implicit function theorem we have

$$\frac{\partial R}{\partial z} = - \frac{\partial \psi / \partial z}{\partial \psi / \partial R} \quad (34)$$

Differentiating and using the fact that  $\psi(R, z) = 0$  we have

$$\frac{\partial \psi}{\partial z} = \left[ \frac{-1}{\theta(c(R, z))} \right] \frac{\partial \log(c(R, z))}{\partial z} + \frac{d \log(w_2(h(R, z)))}{dh} \frac{\partial h(R, z)}{\partial z} - \frac{d \log(v(h(R, z)))}{dh} \frac{\partial v(R, z)}{\partial z} \quad (35)$$

Now we have

$$\frac{z}{R} \frac{\partial R}{\partial z} < 1 \Leftrightarrow -\frac{z}{R} \frac{\partial \psi / \partial z}{\partial \psi / \partial R} < 1 \Leftrightarrow -z \frac{\partial \psi / \partial z}{\partial \psi} > R \frac{\partial \psi}{\partial R} \quad (36)$$

where the reversal in the sign of the inequality comes from the fact that we have shown that  $\frac{\partial \psi}{\partial R} < 0$ . Now note that  $h(\rho t, \rho z) = h(t, z)$  implies that  $R \frac{\partial h}{\partial R} = -z \frac{\partial h}{\partial z}$ . Hence we have

$$\begin{aligned} \frac{z}{R} \frac{\partial R}{\partial z} < 1 &\Leftrightarrow \\ -z \left( \left[ \frac{-1}{\theta(c(R, z))} \right] \frac{\partial \log(c(R, z))}{\partial z} + \frac{d \log(w_2(h(R, z)))}{dh} \frac{\partial h(R, z)}{\partial z} - \frac{d \log(v(h(R, z)))}{dh} \frac{\partial v(R, z)}{\partial z} \right) & \\ > R \left( \left[ \frac{-1}{\theta(c(R, z))} \right] \frac{d \log(c(R, z))}{dR} + \frac{d \log(w_2(h(R, z)))}{dh} \frac{\partial h(R, z)}{\partial R} + \frac{\dot{w}_1}{w_1} - \frac{d \log(v(h(R, z)))}{dh} \frac{\partial v(R, z)}{\partial R} \right) & \end{aligned} \quad (37)$$

Applying  $R \frac{\partial h}{\partial R} = -z \frac{\partial h}{\partial z}$  we have

$$\frac{z}{R} \frac{\partial R}{\partial z} < 1 \Leftrightarrow z \left[ \frac{1}{\theta(c(R))} \right] \frac{1}{c} \frac{\partial(c(R, z))}{\partial z} > R \left[ \frac{-1}{\theta(c(R))} \right] \frac{1}{c} \frac{\partial(c(R, z))}{\partial R} + R \frac{\dot{w}_1}{w_1} \quad (38)$$

$$\frac{z}{R} \frac{\partial R}{\partial z} < 1 \Leftrightarrow \frac{\dot{w}_1}{w_1} < \frac{1}{R} \left[ \frac{1}{\theta(c(R))} \right] \left( \frac{R}{c} \frac{\partial(c(R, z))}{\partial R} + \frac{z}{c} \frac{\partial(c(R, z))}{\partial z} \right) \quad (39)$$

## Appendix 5

**Proposition 4:** When the wage schedule rises the optimal response of the agent is to

(i) keep retirement unchanged and increase consumption proportionately with wages if the utility function has a local inter-temporal elasticity of substitution of unity.

(ii) increase the age of retirement and increase consumption more than proportionately with wages if the utility function has a local inter-temporal elasticity of substitution greater than unity.

(iii) decrease the age of retirement and increase consumption less than proportionately with wages if the utility function has a local inter-temporal elasticity of substitution less than unity.

Proof: Consider an increase in the whole time path of the wage schedule  $w_1$  by a factor  $\tau$ .

We can then write the optimal retirement age and consumption level as function of the parameter  $\tau$ ,

$R(\tau)$  and  $c(\tau)$ . The first order condition (16), which holds for all  $\tau$ , is

$$u'(c(\tau))\tau w_1(R(\tau)) = \pi(R(\tau))$$

where

$$\pi(R(\tau)) = \frac{v(h(R(\tau), z_0))}{w_2(h(R(\tau), z_0))}$$

Clearly, the function  $\pi$  is increasing in  $R$ . Differentiating both sides of the first order condition we can derive

$$\frac{\tau R'(\tau)}{R(\tau)} \frac{R(\tau)\pi'(R(\tau))}{\pi(R(\tau))} - \frac{c(\tau)u''(c(\tau))}{u'(c(\tau))} \frac{\tau c'(\tau)}{c(\tau)} = 1$$

or

$$\varepsilon_{R\tau}\varepsilon_{\pi R} + \frac{1}{\theta}\varepsilon_{c\tau} = 1 \quad (40)$$

Where  $\varepsilon_{R\tau}$  is the elasticity of the retirement age with respect to  $\tau$ ,  $\varepsilon_{c\tau}$  is the elasticity of consumption with respect to  $\tau$ ,  $\theta$  is the inter-temporal elasticity of substitution, and  $\varepsilon_{\pi R}$  is the elasticity of  $\pi$  with respect to  $R$ , which we know to be positive. All these elasticities are local though we suppress their dependence on consumption and age for convenience.

From the budget constraint given by equation (8) we have that the effect of increasing wages by the proportion  $\tau$  is

$$c(\tau) \int_0^T e^{-rt} s(t, z) dt \leq \tau \int_0^{R(\tau)} e^{-rt} s(t, z) w_1(t) w_2(h(t, z)) dt$$

It follows that changes to  $c(\tau)/\tau$  and  $R(\tau)$  have the same sign. Further, from this equation we can derive:

$$\begin{aligned} \varepsilon_{c\tau} = 1 &\Leftrightarrow \varepsilon_{R\tau} = 0 \\ \varepsilon_{c\tau} > 1 &\Leftrightarrow \varepsilon_{R\tau} > 0 \\ \varepsilon_{c\tau} < 1 &\Leftrightarrow \varepsilon_{R\tau} < 0 \end{aligned}$$

Proposition 4 follows from these relationships and equation (40).

## References

- Becker, G.S., T.J. Philipson, and R.R. Soares, 2005, "The Quantity of Life and the Evolution of World Inequality," *American Economic Review* 95: 277–91.
- Bloom, D.E., D. Canning, and B. Graham, 2003, "Longevity and Life-Cycle Savings," *Scandinavian Journal of Economics* 105: 319–38.
- Bloom, David E., David Canning, Rick Mansfield, and Michael Moore, 2007, "Demographic Change, Social Security Systems, and Savings," *Journal of Monetary Economics* 54: 92–114.
- Bloom, David E., David Canning and Michael Moore, 2004, "The Effect of Improvements in Health and Longevity on Optimal Retirement and Saving," National Bureau of Economic Research, Working Paper, No. 10919.
- Carnes, B.A., Olshansky, S.J., and Grahn D, 2003, "Biological evidence for limits to the duration of life," *Biogerontology* 4: 31–45.
- Chang, F-R., 1991, "Uncertain Lifetimes, Retirement, and Economic Welfare," *Economica* 58: 215–32.
- Congressional Budget Office, 2004, *The Outlook for Social Security*, The Congress of the United States, Washington D.C.

Costa, D. L., 1998a, *The Evolution of Retirement: An American Economic History, 1880–1990*, National Bureau of Economic Research Series on Long-Term Factors in Economic Development, University of Chicago Press, Chicago.

Costa, D. L., 1998b, "The Evolution of Retirement: Summary of a Research Project," *American Economic Review* 88: 232–36.

Costa D.L., 2002, "Changing Chronic Disease Rates and Long-term Declines in Functional Limitation Among Older Men," *Demography* 39: 119-138.

Crimmins, E., Y. Saito, and D. Ingegneri, 1997, "Trends in disability-free life expectancy in the United States, 1970–90," *Population Development Review* 23: 555–572.

Cutler, David M., 2001, "Declining disability among the elderly," *Health Affairs* 20 (6): 11–27.

Ehrlich, I., and H. Chuma, 1990, "A Model of the Demand for Longevity and the Value of Life Extension," *Journal of Political Economy* 98: 761–82.

Feldstein, M.S., 1985, "The Optimal Level of Social Security Benefits," *Quarterly Journal of Economics* 10: 303–20.

Fogel, R.W., 1994, "Economic Growth, Population Theory, and Physiology: The Bearing of Long-Term Processes on the Making of Economic Policy," *American Economic Review* 84: 369–95.

Fogel, R.W., 1997, "New Findings on Secular Trends in Nutrition and Mortality: Some Implications for Population Theory," In M. Rosenzweig and O. Stark, eds., *Handbook of Population and Family Economics*, vol. 1A, Elsevier, Amsterdam.

Fries, J.F., 1980, "Aging, natural death, and the compression of morbidity," *New England Journal of Medicine* 303:130-5.

Freedman, V.A., L.G. Martin, and R.F. Schoeni, 2002, "Recent trends in disability and functioning among older adults in the United States: a systematic review," *Journal of the American Medical Association* 288: 3137–46.

Graham, Patrick, Tony Blakely, Peter Davis, Andrew Sporle and Neil Pearce, 2004, "Compression, expansion, or dynamic equilibrium? The evolution of health expectancy in New Zealand," *Journal of Epidemiology and Community Health* 58: 659–666.

Gendell, M., 2001, "Retirement Age Declines Again in 1990s," *Monthly Labor Review* (October) 12–21.

Gordo, Laura Romeu, 2006, "Compression of Morbidity and the Labor Supply of Older People," IAB Discussion Paper, No. 9/2006.

Gruber, J., and D. Wise, 1998, "Social Security and Retirement: An International Comparison," *American Economic Review* 88: 158–63.

Hamermesh, D.S., 1985, "Expectations, Life Expectancy, and Economic Behavior," *Quarterly Journal of Economics* 10: 389–408.

Heckman, James J., 1976, "A Life Cycle Model of Earnings, Learning, and Consumption," *Journal of Political Economy* 84(4): S11-S44.

Hubbard, R Glenn and Judd, Kenneth L, 1987. "Social Security and Individual Welfare: Precautionary Saving, Borrowing Constraints, and the Payroll Tax," *American Economic Review* 77(4): 630-46.

Kalemli-Ozcan, S., and D.N. Weil, 2002, "Mortality Change, the Uncertainty Effect, and Retirement," Working Paper no. 8742, National Bureau of Economic Research, Cambridge, MA.

Kotlikoff, L.J., A. Spivak, and L.H. Summers, 1982, "The Adequacy of Savings," *American Economic Review* 72(5):1056-69.

Laibson, D., 1998, "Life-Cycle Consumption and Hyperbolic Discount Functions," *European Economic Review* 42: 861–71.

Laibson, D.I., A. Repetto, J. Tobacman, R.E. Hall, W.G. Gale, and G.A. Akerlof, 1998, "Self-Control and Saving for Retirement," *Brookings Papers on Economic Activity* 1998: 91–196.

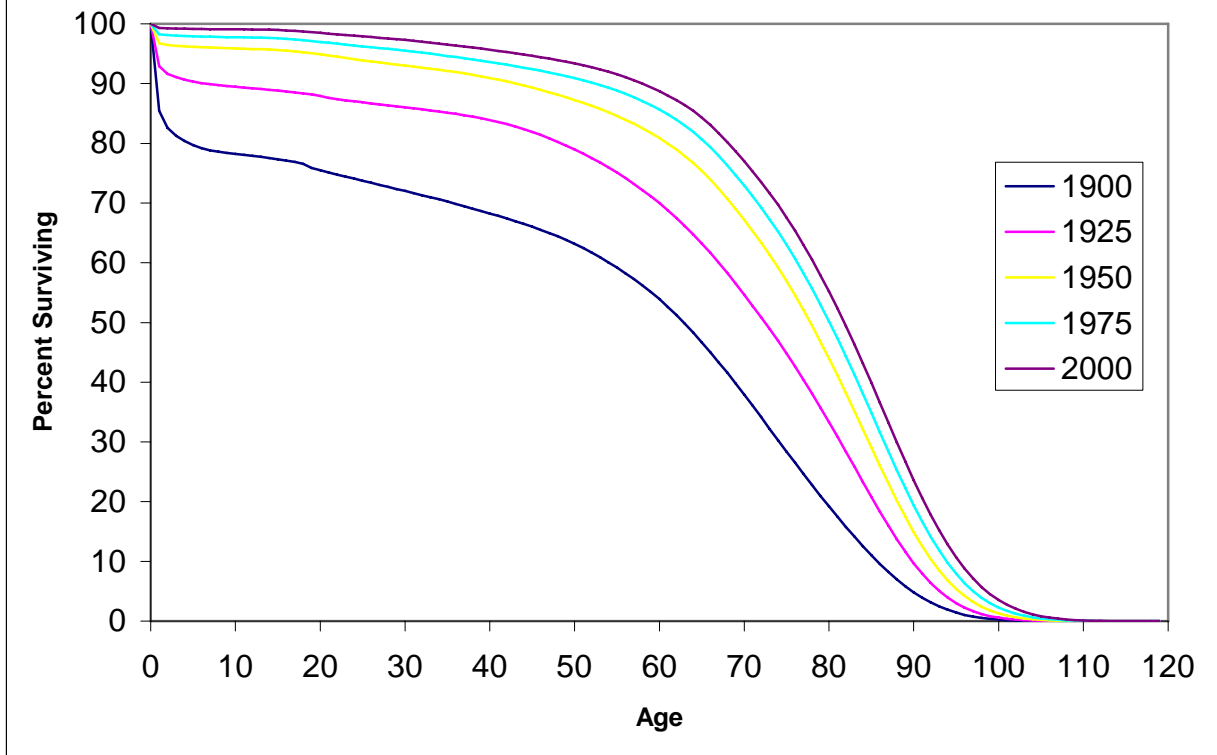
Lee, R., 2003, "The Demographic Transition: Three Centuries of Fundamental Change," *Journal of Economic Perspectives* 17: 167–90.

- Lee, R., and L. Carter, 1992, "Modeling and Forecasting U.S. Mortality," *Journal of the American Statistical Association* 87: 659-671.
- Lee, R., and J.R. Goldstein, 2003, "Rescaling the life cycle: Longevity and proportionality," *Population and Development Review* 29 (Supplement): 183–207.
- Lee, Ronald, and Timothy Miller, 2001, "Evaluating the Performance of the Lee-Carter Approach to Modeling and Forecasting Mortality," *Demography* 38 (4): 537–549.
- Manton, K.G., X. Gu, and V.L. Lamb, 2006, "Long-Term Trends in Life Expectancy and Active Life Expectancy in the United States," *Population and Development Review* 32(1): 81–105.
- Mathers, C.D., R. Sadana, J.A. Salomon, C.J.L. Murray, and A.D. Lopez, 2001, "Healthy Life Expectancy in 191 Countries, 1999," *Lancet* 357 (9269): 1685–91.
- Mor, V., 2005, "The Compression of Morbidity Hypothesis: A Review of Research and Prospects for the Future," *Journal of the American Geriatrics Society* 53: S308–S309.
- Nordhaus, W., 2003, "The Health of Nations: The Contribution of Improved Health to Living Standards," in Kevin H. Murphy and Robert H. Topel, eds., *Measuring the Gains from Medical Research: An Economic Approach*, University of Chicago Press, Chicago.
- Schultz, T.P., 1999, "Health and Schooling Investments in Africa," *Journal of Economic Perspectives* 13: 367-88.

Skinner, J., 1985, "The Effect of Increased Longevity on Capital Accumulation,"  
*American Economic Review* 75: 1143–50.

Sydsaeter K., A. Strøm, and P. Berck, 2000, *Economists' Mathematical Manual*, 3rd ed.,  
Springer, Berlin.

**Figure 1**  
**Projected Survival Rates by Birth Year in the United States**



Source: Berkeley Mortality Database

**Table 1**  
**Testing the Proportionality Condition Relative to the 1900 Survival Schedule**

	Retirement Age in 1900	Year of Survival Schedule				
		1900	1925	1950	1975	2000
Year		1900	1925	1950	1975	2000
Life Expectancy at Age 16		50.7	56.4	59.6	61.8	63.5
Life Expectancy Relative to 1900		1.00	1.11	1.18	1.22	1.25
Initial Consumption- Wage Ratio with retirement age proportional to life expectancy	60	1.14	1.19	1.22	1.25	1.26
	70	1.23	1.27	1.30	1.31	1.32
	80	1.28	1.31	1.32	1.33	1.34

For each retirement age, the proportionality condition is satisfied if the initial consumption wage ratio under the new survival schedule, allowing for an increase in the retirement age proportional to the increase in life expectancy, is no smaller than that with the 1900 survival schedule.

Source: Berkeley Mortality Database and the authors' calculations.