

Converting between Measures of Health Impact:
Health Gap to Health Expectancy

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Abstract

Methods for quantifying health impacts can be categorized in a number of ways. When measuring impacts in life-years, two categories predominate, namely Health Gap (HG) and Health Expectancy (HE) measures. HGs follow a cross-sectional, static, approach for calculating impacts, tallying the number of years of life lost accruing within a specified span of calendar years. In contrast HEs use dynamic models (such as life-tables) to track the differential prognosis of two hypothetical cohorts (exposed versus unexposed) across their life-courses. Despite their distinct methods of calculation, HG and HE have the same goal (a summary of impact) and draw upon the same input data. HGs and HEs can therefore be thought of as two different scales, for measuring the same thing. As such, they are akin to degrees Celsius and Fahrenheit as alternative scales of temperature. Yet unlike the case of temperature (where well-worn conversion formula exist), there is no known formula for converting between HG and HE measures. In this paper we marshal a typology of HG and HE approaches to support our effort to define such conversion formulas. We derive (and test) our (approximate) conversion formulas, based on a period life-table construct, restricting attention to summaries of mortality alone. We test the performance of conversion formulas using data from the HMD database (<https://www.mortality.org/>). The conversion formulas, which are to the best of our knowledge new, should prove useful enabling conversions of impact estimates between the two approaches, and in better understanding the circumstances under which impacts obtained under the contrasting scales will converge/diverge from each other.

Keywords: Life-table; excess rate ratio; life-years lost; Cross-sectional measures; demography; Health Impacts; Burden of Disease; Comparative Risk Assessment.

1 Introduction

The population health impacts associated with heightened or dampened exposures to risks such as air-pollution have been quantified in the literature using a wide array of yardsticks. A key distinction when outlining a taxonomy of these alternative yardsticks is that between those that accentuate the positive, tallying up what might be referred to as “blessings”, versus those that accentuate the negative, tallying up what might be referred to as “losses.” In short the summary of impact can report a change in “blessings” or a change in “losses” as the bottom-line summary of impact. When the numeraire of health is time-based this distinction between counting blessings or losses is marked out by Health Expectancies (HE) — blessings — versus Health Gaps (HG) — losses.

HEs and HGs are understood to be qualitatively related [1, 2, 3], but to date results produced by the two approaches have been treated as sufficiently distinct so as to preclude a quantitative conversion from one to the other. On the surface the mathematics expressing the two measures, shows little promise for establishing an algebraic path for translating between the two [4, 2]. This paper presents a systematized effort to establish such a relationship, and reveals that a key construct underpinning the more complicated of the two scales (the health expectancy scale) is in actual fact a variant of HGs — this component marks the outline of a proverbial *masterkey* for enabling the sought-after quantitative link, but other distinctions require careful attention as well.

The resulting algebraic link between the health-gap and health-expectancy impact scales, reveals the importance of normalization, age-standardization, and two other distinctions when trying to equate the two scales. We demonstrate that our proposed algorithm for converting between HEs and HGs performs well when appropriate steps are taken to assure comparability with respect to the aforementioned distinctions; indeed these steps of assuring comparability embody our algorithm.

To maintain a manageable focus and to provide a strategically well posed proof of concept, we restrict our study in several ways. First, we examine time-based summaries of mortality impact alone (choosing to leave for future study, summary measures that account for morbidity). That is, our focus in this paper is exclusively on quantity of life, as determined by mortality [1, 4]. Moreover, while exposure induced perturbations of a particular cause of death (e.g., cardiovascular disease related death) are admittedly often of interest, our analysis focuses exclusively on the perturbations to the all-encompassing category of “all-cause” mortality. And while perturbations can encompass the dampening of the status-quo mortality rates, we restrict our attention to those that amplify the rates. Finally, we examine the special case where the *counterfactuals* [5, ?] of interest cause a fixed multiplicative change in the age specific all-cause mortality rates that is invariant with age. So for example, we might be interested in the health impacts that come with a counterfactual [5] that amplifies the mortality rate in every age interval by 10% — this will be referred to herein as an excess rate ratio of 0.10 or $\varepsilon = 0.10$ (the excess rate ratio can be understood the proportional change in age-specific mortality rates that defines our perturbation of interest).

2 Background

2.1 Measures of Health Impact

A key appeal of summary measures of population health, is the role they serve as a common currency (or yardstick) that is useful for ...

- Juxtaposing risk-factors (*wrt* their health impacts)
- Supporting health economic evaluations (e.g., CEA, BCA), and
- Reporting impacts in salient terms

For illustration of the third point, consider a 1 percent increase in the absolute mortality rate. We argue that such a summary has very little salience, and further argue that a more readily understood summary would express the ramifications in terms of life-years lost, say a 0.1 year of life lost.

2.2 Overview of Health Gaps

A survey of HI measures that use HGs as a basis, essentially amounts to an overview of baseline HG measures, because the impact is simply calculated as a difference between the status quo measure and a counterpart that is upwardly scaled to capture the amplified mortality rates. Health-gap measures essentially total up the number of person-life-years by which a population’s health-state falls short of a stated norm. Following the proverbial “glass half empty” metaphor HGs amount to quantifying the degree to which the water-level in the glass falls short of its top (analogue being a ‘full life’).

Perhaps the simplest of the health gap measures identifies a maximum life-span (popular choices over the last half century have been spans of 65, 75, or 85 years, but others have been chosen). Under this approach, which goes under the label of a PYLL [6, 7], a loss would be calculated as $L - x$ (where L is the maximum life-span chosen for the analysis). So for example, a death occurring within the age-interval [25, 26) would be assigned a loss of $L - 25.5$ (i.e., a loss of on the order of 60 years for an L of 85). By convention this approach does not count deaths that occur beyond the chosen span, L . The convention meets (not surprisingly) with controversy, with many uncomfortable with leaving deaths (that occur post L) uncounted. There are alternatives to the PYLL that side-step (at least in part) this contentious choice by refining how the normative goal (for a good life) is defined. The HG measure called the person-expected-year of life-lost (PEYLL), does this by invoking the age-indexed schedule of residual life-expectancy so as to articulate an age-dependent norm for a good life. It is the PEYLL that will be our focus when

contrasting HG measures against their HE counterparts.

Residual life-expectancy is a generalization of the well-known summary, life-expectancy at birth (e_0^o); where the calculation generalizes to allow conditioning on any age of interest. We refer to the locus of e_x^o estimates computed for the midpoints of each successive age-interval as the *residual life-expectancy schedule*. By way of illustration, for an analyses using single year age intervals, the 50th element of the *schedule* would reveal the residual life-expectancy of someone who is 50.5 years old. A reasonable value would be around 36 years ($e_{x=50.5}^o = 36$). Someone dying at a later age would be ascribed a smaller shortfall, in accordance with the residual life-expectancy schedule e_x^o . So for example, $e_{x=85.5}^o = 8$. Note that under this scheme for defining the norm for age dependent good life-span, all deaths, even those occurring well after the populations life-expectancy, will be accorded a nonzero loss. As others (cf Murray [1, 8]) have pointed out, ethical questions can be raised about the appropriateness of the PEYLL when drawing comparisons across the world.

2.3 Overview of Health Expectancy Measures

Health expectancies capture the number of years that a person could expect to live in full health [9, 10]. While a key feature of these methods is their ability to capture both quality and quantity of a life-span, our application herein will focus exclusively on their use when quantifying the quantity (duration) of an expected life-span, subject to the mortality rates that apply to them. The summary measure life-expectancy (at birth) is an example of the type of health-expectancy measure that we will be relying upon. This measure relies upon conditions documented in a contemporary calendar year (we will be using the year 2005) for assessing the status-quo conditions. In particular the mortality rates estimated as prevailing in that calendar year are used to compute the probabilities of surviving from one age-interval to the next. This so-called *period* approach contrasts with a *cohort* approach that attempts to predict what the mortality rates will

be at advanced ages in the calendar year when logic indicates those ages will be reached. The period approach by-passes this need for forecasting by pragmatically assuming that the mortality rates facing say a 60 year-old in the contemporary calendar year, will provide a reasonable proxy for the rates facing the 60 year-old in the future calendar year when a birth-cohort would age into their 60th year. In this work we will focus upon *period* approaches to calculating life-expectancy and in particular changes in life-expectancy that are implied by changes in the mortality rate schedule faced by a population.

2.3.1 Normalization and Standardization

HG measures like their HE counterparts can be normalized in one of three ways, namely in per country/population terms (no normalization), in per person-year terms (akin to a mortality rate), and in per death terms [11, 12, 13]. The age-structure can be dealt with in one of several ways (here we address just three, but there are others), namely in crude (no adjustment for age structure), in directly standardized terms (making use of the direct standardization approach), and in implicit life-table standardized form (indirect standardization might present one of the other options that we leave aside in this work). See Table 2 for summary of variants considered under our N (normalization) and S (standardization) taxonomy.

2.4 Problem Context

In the realm of quantifying the health impact of Air-pollution, as in other realms, some analysts favor a HG approach (for example, change in person-years of life-lost, PEYLL, has been a popular choice) for quantifying impacts, while others favor health expectancy (for example period life-table summaries of life-years lost are popular) [14]. For convenience we will refer to these two different approaches as the cross-sectional (i.e., illustrating a HG approach) and the period (illustrating a HE approach) approaches.

	“Cross-sectional” Approach	Period (Life-table) Approach
Impact	15000 person years of life lost	0.1 years of life expectancy

Table 1: Impact of Air-pollution projected for Canada based on 2005 vital and census statistics, a shared exposure response relationship, and estimates of air pollution exposure in Canada.

To illustrate, these separate approaches might be applied to summarize the impact of air pollution in Canada (using 2005 data), amongst females by following a few steps. Assuming an excess rate ratio, ε , of 0.01 we would obtain the cross-sectional impact as follows.

$$\begin{aligned} \text{YLL} &= (\sum D_x e_x) \varepsilon \\ &= (1480,000) 0.01 \end{aligned}$$

where D_x represents the death-count in the age interval $([x, x + 1))$ and e_x represents the residual life-expectancy ascribed to someone dying at the midpoint of the aforementioned age-interval.

As summarized in Table 1 these two estimates are clearly not on the same scale. Even though they differ numerically, they might still be compatible with one and another. Is it possible to demonstrate that 15000 person-years from the cross-sectional approach is equivalent to 0.1 years/person from the life-table approach (see Table 1), in the same way that 32° F is equivalent to 0° C? In this paper we investigate if it is possible to convert between the numerical results obtained under these two approaches (the cross-sectional and the period life-table approaches).

As we study the distinction between HGs and HEs we shall find the schematic in Fig 1 helpful. The schematic displays a curve that represents the fraction of a birth cohort that remain alive as a function of their attained age — a curve known as the *survivorship*. This curve is helpful for contrasting health expectancy and health gap measures. They share a complimentary

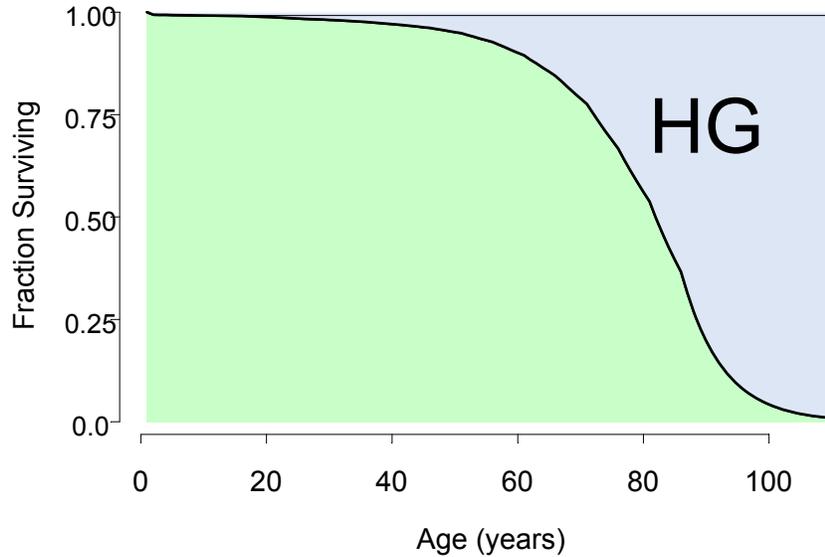


Figure 1: Schematic plots the *survivorship* function against age. The area below the survivorship function is of interest for computing Health Expectancy measures, whereas the area shaded blue is of interest for computing health gap measures (see HG label).

relationship; where health expectancy measures tally up gains (focusing on how full a glass is); health gaps tally up losses (focusing on how far the glass falls short of being full). The schematic depicts one version of a health gap measure with the blue shaded area representing the degree to which the survival experience of the birth cohort fell short of a scenario in which all survive until an upper bound age which might be represented in this case as age 110 — this corresponds the aforementioned PYLL. In our analysis we use a different variant of HG namely the aforementioned PEYLL, which is differentiated from the PYLL (shown in the stylized figure) by a different norm from which losses are calculated. The compliment of the blue area (shaded in green) represents the number of years of life that the birth cohort will enjoy, and depending on the details of calculation can represent the life-expectancy of that birth group (as it will do

in our own work).

The period life-table approach directly instantiates a health-expectancy type of measure, whereas the cross-sectional approach instantiates a health-gap class of measure (albeit using a different nominal reference life-course than represented in the schematic). The two measures differ from one and another in several other respects that are worth reviewing.

The characteristics of the two Impact Measures are contrasted below. The distinctions turn on two simple attributes of population health measures (Table 2). The first of these addresses the question of *normalization* which refers to an effort to express impacts in a per “something” basis. The most common distinction here would be whether counts (of either gains or losses) represent totals across a whole population, or whether they represent the average (*per-capita*) impact. These two variants, namely un-normalized and normalized in per-capita terms will be useful in comparing two impact summaries, while a third variant of normalization proves to be central in bridging the two impact measures — this third variant expresses counts in what we might refer to as *per-affected-capita* as opposed to *per-capita* terms. An example from a different realm might serve to clarify per-affected capita normalization. Consider the average coffee consumed per capita per day, and note how higher estimate would apply if expressing average coffee consumed per coffee-drinker (our analogue for per affected-capita). In this paper we shall find it particular useful to consider per-extra-death normalization (determining the loss, in years, per extra death implied by the perturbation).

The second of the attributes refers to an effort to age-standardize counts [15, 11, 12, 16]. In this work we highlight three age-standardization approaches, with the first approach actually representing “no standardization” (such counts are often referred to as *crude* in efforts to distinguish them from their standardized counterparts). Those acquainted with health data are likely familiar with the role of age-standardization when summarizing age-stratified mortality rates with a singular summary statistic. The method of choice for computing this summary

Normalized (if at all)	Age-standardized (if at all)
N0 Un-normalized	S0 Un-standardized (Crude)
N1 Per-capita normalization	S1 Direct age-standardization
N2 Per(-attributable) death	S2 Life-table (implicit) standardization

Table 2: Table proposes a taxonomy for distinguishing Health Impact measures based on: choice of normalization (first column); and age-standardization (second column). Normalization refers what if any effort has been made to scale impacts to a common denominator. Choices include none (expressing countries impact in total, aggregate, terms), per capita (N1), or per extra (caused by the perturbation, ε) death (N2). Age-standardization choices considered include none (S0), direct age-standardization (S1), or the age-standardization that is implicit to life-table calculations. In practice, there may additional choices not examined here.

is the direct method, which yields the summary referred to as *age-standardized mortality rates (ASMR)*. While age-standardization is most commonly practiced in relation to mortality rates, the same objectives (ruling out age dependency) can motivate the age-standardization of other summaries [15, 11, 12, 16].

We introduce nomenclature (Table 5) to label the different versions of normalization and age-standardization that are relevant to the Cross-sectional and Period measures. This proves to be helpful in our efforts to find an algorithm for converting between the to HI variants.

With these distinctions in place, let us review where the most customary versions of the cross-sectional and period summary measures are situated in relation to this emerging taxonomy (differentiating by normalization, standardization, as per Table 2 and additional attributes that

Method Attribute	Cross-sectional	Period
HG/HE measure	HG	HE
Normalized (N#)	No (N_0)	Yes (N_1)
Age-standardized (S#)	No (S_0)	Yes (S_2)
Static/Dynamic	Static	Dynamic
Counting horizon	snap-shot	life-course

Table 3: Table lists the key distinctions that differentiate cross-sectional and period (time-based) measures, highlighting the the authors judgment of the most customary normalization (N) and standardization (S) choices that are made when using these measures. The last two rows identify additional distinctions, noting, in turn, that cross-sectional measures are static and thus ignore dynamic feedback, and that a different *counting horizon* is implicit to the cross-sectional versus period based approaches.

are yet to be introduced).¹

As summarized in Table 3 the conventional Cross-sectional measure is neither normalized nor standardized. In contrast Period (herein calculated using the life-table algorithm) measures are implicitly normalized and standardized. The period summary measure (familiar to lay-people as a life-expectancy) is understood to be a per-capita measure (though we will have reason to look more closely at this apprehension). The life-table algorithm confers an implicit age-standardization (one we refer to using the N_2 notation). So in summary, the period constructs are normalized in per-capita terms (N_0 in our nomenclature) and implicitly standardized (S_2 , to indicate standardization via life-table algorithm).

Table 3 introduces two additional attributes namely “static/dynamic” and “counting hori-

¹Interestingly these differentiating attributes typically go unstated (implicit to the choice of measure, unless otherwise stated) in reports that use them.

zon” that are also useful for distinguishing the cross-sectional and period measures. These are described in turn. The “static/dynamic” distinction differentiates the two approaches based whether they account for feedback between successive time-intervals. In the case of cross-sectional approaches, a calendar year time-frame (typically a single year) is conceptualized and no feedback between successive time-intervals is considered.² In contrast the period approach integrates its event count across time in a manner that is overtly attentive to feedback³ The distinction between static (cross-sectional) and dynamic (period) approaches is paralleled by the level of mathematical complexity in the expressions required to express these two approaches.

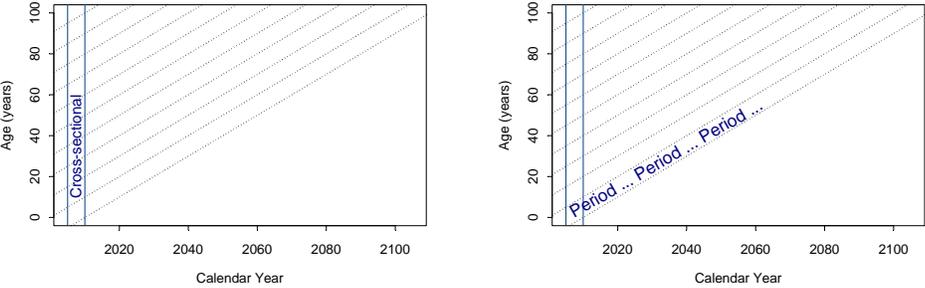
The attribute labeled “counting horizon” refers to the boundaries delimiting what events (deaths, or more specifically extra-deaths, in this case) get counted. Two time dimensions are recognized as being helpful for describing the prognosis of a population: instructively depicted in a Lexis diagram, which shows *age* (on the vertical axis) and *calendar year* (along the horizontal axis). Cross-sectional measures represent a snap-shot in time, which is represented in the Figure as a vertical time interval. This time interval is most commonly one year in duration (horizontal width). It has been depicted in Figure ?? as a wider interval to accentuate visibility on the display. Cross-sectional measures sum up all deaths for the country of interest that occur with this narrow calendar year time frame (typically a year).

In contrast, the period measure, in our case life-expectancy, sets boundaries over what counts differently. Life-expectancy purports to summarize the experience of single birth cohort (defined by the calendar year in which the birth cohort members are “born” — theoretically these all are

²While multiple age-strata are considered the event count taken within each age-strata is effectively a snap-shot within the frame defined by the cross-section

³Capturing, for example, the reality that the number of deaths in the $(i + 1)^{th}$ interval are dependent upon the number of deaths in the interval immediately preceding it, namely the $(i)^{th}$ interval; since the deaths in the preceding interval determine, in part, the number of people alive and subject to the force of mortality in the, $(i + 1)^{th}$ interval.

conceptualized as being born simultaneously), and then focuses its count on the experience of that particular birth cohort as they move through their life-course. The count is extended until the birth cohort (theoretically) exhausts itself (the last individual dies). Thus the boundaries on what counts are represented by a diagonally oriented trapezoid/rectangle; with the diagonal following a one-to-one line ⁴. An implication of these distinctly different boundaries of the *counting horizon*, is the different amount of calendar time over which a count is extended — a different count horizon. While the count is typically extended (in the calendar dimension) by just a year, it is extended in the period construct by calendar time frame that is roughly 100 times that time-span. This manifests in a different estimate of extra deaths attained by the cross-sectional versus a (naive adaptation) of the period measure.



(a) “Cross-sectional” Approach

(b) Period (Life-table) Approach

Figure 2: Lexis diagrams used to highlight distinction about ‘what counts’ between the cross-sectional and period approaches.

⁴The diagonal in a Lexis diagram reflects the constraint that each year that the birth cohort progresses through their life-span must be matched with year of progress in the age dimension; perhaps more simply put, each additional year lived by the birth cohort, marks in increment in age of a single year (for those remaining alive that is)

3 Methods

3.1 Data

Our analysis requires age-stratified count data, by country, capturing deaths (D_x) and the population (P_x) at risk. We also make use of life-tables for the same countries. The Human Mortality Database (HMD; (<https://www.mortality.org/>)) provides a convenient repository of the count and life-table data for 49 countries; indeed providing such data separately for multiple calendar-years. In this work we data for the period of 2005 as our focus examining the country specific summaries; choosing 2005 because it was one of the more recent periods where the data coverage was complete for all 49 countries. We restricted attention to counts and life-tables that were provided at the maximal resolution for age-intervals: namely data for single year age intervals. The HMD data are stratified by gender as well, with categories of: females, males, and the category “both” representing females and males combined. Since we did not anticipate any materially different patterns across gender categories, we chose to restrict our analysis to females. Data last downloaded on July 2019.

In the case of age-stratified population counts, we rely on the HMD files labeled **Exposures** (which refers to “person-time at risk of death”), but we choose to label the age-stratified values as P_x noting that they are with few exceptions identical to their age-stratified counterparts in the files labeled *Population*. We note that population counts (i.e., whether from the **Exposures** or *Population* files) will be small and indeed occasionally zero for the age intervals that can be described as exceptionally old (e.g., age interval [107, 108)). To avoid “*divide by zero*” errors we have chosen to drop age intervals beyond age 105 from all countries.⁵ The issue had no bearing on

⁵Noting that the time-based summaries of interest to this work (summed across all age intervals) are known to be comparatively insensitive to the details of the exceptionally later age intervals, we are confident that the summaries relied upon for our analysis are robust to the choice to drop age intervals post 105.

our period (CMLT [4]) calculations, and so we kept all age intervals for the life-table calculations.

6

We briefly outline our calculations for the cross-sectional, period, and related constructs (such as the crude mortality rate).

By convention age-specific death counts for single-year age intervals are expressed D_x connoting the count occurring within the age interval $[x, x + 1)$, where that count understood to apply to a period (herein, 2005) and a specified country of interest. So for example D_{10} connotes the count for the age interval $[10, 11)$. Similar conventions are followed in regards to age-stratified population counts (P_x) and mortality rates (M_x).

We adopt a similar convention for the standard life-table constructs, describing age interval specific constructs like the life-table probability of dying q_x , the life-table deaths (d_x), and the life-table (stationary) population counts L_x [17]. Other life-table constructs describe properties that apply to a specific age (rather than the aforementioned that summarize an age interval) and these are denoted similarly. We refer the reader to Refs ?? for more detail about life-table calculations and notation.

The calculations for cross-sectional constructs are summarized in Table 4 using the YLL to denote the PEYLL HG variants along with their life-table counterparts. All variants can be expressed as a scaled version of a weighted average calculation where the scalar multipliers depend on the normalization variant (N) and the variant of standardization (S). The weighted average is a weighted average of the residual life-expectancy schedule for a country (denoted e_x^o) in all four cases (indexed by S) but involves different weights depending on the variant of age-standardization (see table). No scalar multipliers are required for the N2 (per-excess

⁶While this minor disjuncture in the calculations for the period (life-table) versus the cross-sectional (which were subject to the truncation of intervals beyond 105) is not ideal, we argue that it has had no material impact on our findings.

	S_0	S_1	S_2	life-table
Weights ($w_i \propto$)	D_x	$M_x P'_x$	$M_x L_x$	d_x
$N_2 \rightarrow N_1$	CMR_ε	$ASMR_\varepsilon$	$ASMR^{Lx}_\varepsilon$	LR_ε
$N_2 \rightarrow N_0$	$D_T \varepsilon$	$D'_T \varepsilon$	$D_T^{Lx} \varepsilon$	$P_T^{Lx} ASMR^{Lx}_\varepsilon$

Table 4: Health impacts measured as changes in a HG measure (ΔYLL) can be calculated with reference to this entries in this table. An N2 variant of ΔYLL (namely expressed in *per excess death* terms) is easily calculated, for a country, as a weighted average of its residual life-expectancy schedule, where the choice of weights depends on the S variant of interest (by column). For a given S variant of choice, one can convert from the resulting N2 measure to its N1 or N0 counterpart by employing the S-variant specific scalar multipliers as identified in the appropriate column and row of this table.

death) normalization (as shown in Table), thus demonstrating that the N2 HG constructs are simply the aforementioned weighted-averages of the residual-life-expectancy schedule, and can be understood to provide some measure of the typical number of years lost per death, under the population-age structures operative for each S variant.

The weighted average of e_x^o using the weighted specified in Table ?? yields an N2 (per excess death) health-gap variant. Here is where we see the kernal of commonality between a life-table and a cross-sectional approach. In particular, the N2 measures labeled S2 and life-table can be shown (subject to reasonable conditions) to be identical to one and another. Our purpose in listing the life-table summary (with weights d_x) is central to this paper — this N2 measure coincides with a measure that is identified in the Demography literature as e^\dagger . It embodies the “masterkey” that we alluded to earlier, since it plays a central role in a first-order approximation of the CMLT calculations of ΔLE (our so called period construct). Specifically, as Keyfitz and others [18, 19, 20, 4] have indicated,

$$\Delta LE \approx e^\dagger \varepsilon \tag{1}$$

which as per our discussion above, applies in the case of multiplicative amplification/dampening (as per ε) of the all-cause mortality rate. In words it suggests that the change in life-expectancy implied by say a 10% increase in the all-cause mortality rates (applied to all age-intervals) would imply a change in life-expectancy that is approximately 10% of e^\dagger . A typically value for e^\dagger is around 10 years, thus indicating a change in life-expectancy of around 1 year. Remarkably the e^\dagger playing this central role in approximating LYL is readily identified as an N2 cross-sectional measure. Playing a role in **both** the period and cross-sectional measures it is the Masterkey that will allow the two approaches to be bridged.

As Brand [4] has demonstrated the approximation (Eq 1) performs well over a wide range of practice. The work herein reaffirms this finding suggesting an arguably tolerable relative error, namely on the order of 10%, provided that ε remains in absolute value terms below 0.25. It is worth pointing out that there are few risk-factor and exposure combinations that would imply a perturbation of all-cause mortality that even approaches such an excess rate ratio.

In summary not only can we identify the period measure (at least its first order approximation) as a HG measure, but this HG variant can be understood as simply a special case of S1 (that is direct age-standardization) where the referent population is chosen to be proportional to the L_x constructs from the life-table. It is this observation that will inform our proof of concept test.

The weighted averages,

$$E_w[e_x]$$

with appropriate choice of weights (see Table 4) will yeild an N2 summary measures. For the S variant of choice (see weights row in Table 4), the other variants can be obtained from those

n2 measures by invoking the appropriate scalar multipliers as identified in Table 4. In the case of translating from an N2 to an N1 (per capita) measure the scalar multipliers not surprisingly involve relative frequency measures, namely crude, standardized rates or life-time probability of dying: with the CMR_{ε} being indicated for S0, $ASMR_{\varepsilon}$ for the S1 case, and LR_{ε} for the life-table case.

In essence this scaling to go from an N2 to an N1 measure can be understood as completing steps, first converting the N2 measure into a total summation of health gaps across all excess deaths (numerator of relative frequency measure) and then in second step dividing by the size of the pool at risk (denominator). For example in the S0 case this scale entails CMR_{ε} which is equal to $(D_T\varepsilon)/P_T$.

When going from an N2 measure to an N0 measure the scaling factor amounts to an expression of the excess deaths implied. In the case of S0 this amounts to $D_T\varepsilon$.

We use the notation Δ to denote change or difference (herein referring to the difference in a population's health between two conditions, namely: (1) status quo conditions; and (2) exposed conditions). For example, using the notation YLL as short form for the PEYLL calculation, we use the notation $\Delta YLL_{(N0,S0)}$ to refer to the difference in the standard cross-sectional measure (a health gap measure) taken between the status-quo, unexposed, and the exposed. This expression can be expressed as the impact among the exposed (simply the status quo health gap multiplied by $(1 + \varepsilon)$) subtract the status-quo $YLL_{(N0,S0)}$. This expression simplifies to $YLL_{(N0,S0)} \times \varepsilon$. Similar notation will be used for the period summary of health impact. In the absence of morbidity, this measure amounts to a difference in life-expectancy (that enjoyed under the status quo less that enjoyed under the exposed conditions. We use LE as short form for life-expectancy (to be calculated from the life-table) and ΔLE to connote change in life-expectancy. Since change in life-expectancy measures obtained from the 'Cause Modified Life-Table' algorithm (see for example [4]) is inherently an N1 and S2 measure, we might augment our notation as follows,

$\Delta LE_{N1,S2}$ (see Table 2).

3.2 Adjustment for Dynamics

Health-expectancy methods such as life-tables or Markov models, owe some of their comparative complexity to their explicit effort to account for dynamic feedback between. In contrast HG measures (including PEYLL) ignore this issue and thus arguably over-count impacts. In particular, if the estimates they produce are interpreted as foretelling the change in health that would repeat in successive years of a time-horizon, then they most certainly are overstating impacts. For our purposes, if we are to obtain a fair comparison of our cross-sectional *DYLL* with its period counterpart *DLE* then we would like an ability to adjust the cross-sectional result downwards to capture the dynamic feedback. We refer to this adjustment as AF_{dyn} . Here we briefly examine this issue, drawing on the results of a previous effort [21]. Brand [21] proposed a simple expression for allowing adjustment from an approximation of ΔLE to its exact counterpart. Using the notation *E* and *A* to connote *exact* and *approximate* estimates of ΔLE , the proposed adjustment factor was

$$\begin{aligned} AF_{dyn} &= \frac{E}{A} \\ &= \frac{1}{1 + 0.40\epsilon} \end{aligned}$$

We shall be interested to see if this approximate adjustment serves us well in the new context of this analysis (2005 data rather than year 2000 data, and a more narrow coverage of countries in the HMD presented herein).

Count, rate, and life-table Data from 49 countries have been extracted from HMD for females, the period 2005, and single-year intervals. The data have been used to compute N1 ΔYLL measures and their period counterparts (ΔLE) along with mortality rate summaries (CMR and

ASMR). In the case of the age-standardization protocols, two referent populations have been used, including the WHO standard [22] and one obtained as the average of the 49 countries L_x schedules (marking out the stationary populations for their respective life-tables) [17]. The latter (S1.ELx) age structure is typical of developed countries whereas the WHO referent more broadly representative of all WHO countries (a systematically younger population).

4 Results

Following the concepts and methods laid out in the previous section we will formulate several paired data-sets, pairing N1 Δ YLL summaries (or variants thereof) with their life-table (Δ LE) counterparts (or variants thereof), where our reference to *variants thereof* is our placeholder for the proposed adjustments for the distinctions laid out in Table 3. To maintain a manageable focus in our primary and first phase of analysis, we compute HIs for a deliberately small excess rate ratio ($\varepsilon = 0.01$). Our purpose is to rule out the need for adjusting for dynamics (an issue that causes material distinctions only when larger excess rate ratios are used). We will close out our analysis with a secondary wave of analyses that explores performance under larger values for ε .

Scatterplots are used throughout our analysis to examine the correspondence between our HI measures. We use the same convention throughout, assigning the cross-sectional measures (with/without adjustment) to the role of an explanatory variable (x-axis) and its period counterpart (Δ LE) to the response variable (y-axis). We typically superimpose a one-to-one line on the scatter-plots as a reference: as the alignment of plotting points gets closer to this line, the match between our two measures improves, and the closer we are to revealing a viable conversion algorithm.

We begin with S0 (crude, non-age standardized) variants of the cross sectional summaries.

Fig 3 reveals that the per-capita version of ΔYLL measures are not in the same scale as their period (ΔLE) counterparts. The ΔLE plotting points are on the order of 100-fold greater than their ΔYLL counterparts, and the vague positive correlation is quite disappointing. A more promising relationship is revealed when we adjust the ΔLE results for the *counting horizon*. When using S0 ΔYLL for the explanatory variable, this adjustment amounts to multiplying the period results by the CMR. Fig 4 shows the clear benefit of this adjustment. Not only are the two variables (response and explanatory) now covering roughly the same range of values, but the positive correlation is now strikingly evident. A bias is evident, with ΔYLL overstating its (adjusted) period counterpart, a bias that appears to decrease (possibly becoming negligible) as the magnitude of HI increases (the higher end of the HI domain is likely predominated by countries with larger CMRs). The absence of an adjustment for dynamics accounts for very little of this bias (results not shown), an observation anticipated having purposely chosen a small value of ε . While the correlation is indeed striking the bias is rather material and begs for an explanation. In previous work [23] we had posited that age-standardized (S1) ΔYLL measures would be better matches for their period counterparts. That work seemed to confirm this for a larger more diverse set of countries (194 WHO countries), albeit with just one referent population. To explore this question further, we examine two S1 variants of the ΔYLL measure; one using the average L_x schedule as the referent population (labeled a) and the other using the WHO's referent (labeled b).

Fig 5 superimposes two additional series along with the series shown in the previous Figure (Fig 4). The series labeled S1a computed the explanatory variable ΔYLL following direct standardization with the average L_x schedule as the referent country, while that for the S1b series used WHO as the referent country. These S1 series require a different adjustment for the period result (response variable); rather than multiplying by the CMR (as was done for the S0 series) they involve multiplying by an ASMR, with the ASMR being calculated using the appropriate

referent (average L_x for S1a and WHO for S1b). While both of these S1 series seem to demonstrate an improved for a subset of the HI domain, that, in both cases, appears to come at the expense of poorer performance in other parts of the domain. We defer for future investigation the question of what explains the differential performance patterns across the HI domain for future investigation. For the purposes of this paper, our priority is to test our *proof of concept*. Before turning our attention to that task we wish to provide some affirming evidence that the bias in relationship between our cross-sectional and period constructs might be traceable to differences in the age-structures underpinning the two calculations. To partially examine this question we revisit the data in Fig 4, parsing the series into three equal (and partly overlapping) subsets that separate the data by the average age of the population underpinning their cross-sectional results. The graph reveals that countries with an older average age (typically more developed) tend to be higher in the HI domain (for S0 case), and tend to reveal better matches between the cross-sectional and period constructs. This result which may on the surface seem counter-intuitive, since one might have expected the more developed countries to have lower HIs, is likely to be driven by the higher CMRs that could arise in developed countries owing to confounding by the population age-structures (tend to be older).

We now turn to our proof of concept: namely our effort to demonstrate that an identity can be achieved between cross-sectional and period constructs. As argued in the methods, a unique application of age-standardization, namely assigning what might be referred to as an internal referent age-structure to each country, namely their respective (country specific) L_x schedules (as opposed to the average of these 49 schedules that was used above under the S1b series) ought to achieve the identity. Fig 7 tests out this proposal. The age-standardization strategy (using each countries life-table L_x schedule as its referent population) succeeds, providing a near identical match between the cross-sectional and period constructs. A very small, yet consistent, bias is evident revealing that the cross-sectional results overstate their period counterparts; but again

by a small margin. Analysis not shown has demonstrated that our approach for providing an approximate adjustment for dynamics to the cross-sectional measure (this acts to dampen its result) addresses roughly half of this bias.

To this point we have yet to address the adjustment for dynamics. This adjustment would (in the case of a positive ε) scale the cross-sectional result downwards to account for the negative feedback that is captured by the period life-table. Brand showed previously that the adjustment AF_{dyn} is directly related to the relative error that would be computed by comparing the exact and approximate methods for computing DLE (based on the CMLT paper and approximations thereof [18, 4]). To test whether the adjustment offered in Brand [21] would serve us well in this context, Fig 8 has been produced based on the Canadian Female, 2005, life-table data. The result suggests that the adjustment works reasonably well.

5 Concluding Remarks

The key distinguishing attributes between cross-sectional and period based life-year health impact measures have been summarized. The blessings versus losses distinction by itself does not pose as much of an impasse to establishing equivalence as may first appear. By exploiting an approximation of the period based measure we have shown that behind a health-expectancy measure of this sort sits a health-gap measure. Since the period measure has a health-gap measure as a kernel of its expression (albeit approximate), it was possible to translate between these two categories of health impact measures. Other attributes seem to demand adjustments if we are to establish a quantitative bridge between these two measures. First, the attribute of normalization requires attention. While cross-sectional measures are most naturally un-normalized, period measures are normalized in per capita terms. We find that third form of normalization (in per perturbed or “attributable” death) serves as a useful bridge between the cross-sectional

and period approaches; indeed the period counterpart of this measure is the ‘kernel’ health-gap measure to which we referred above. Normalization is indeed an obvious attribute to attend to if one is to be sure of comparing like with like measures, but three other attributes were established to be influential in this work as well. Of these, the *counting horizon* appears to be the most substantive adjustment. Whereas attributable deaths (under an N0 variant) scale in proportion to the total annual deaths in a target population when using a cross-sectional measure, they scale in proportion to the prevailing total population size when using a period measure (that is when formulating both cross-sectional and period measures in non-standardized, N0, terms). The last distinguishing feature examined herein, is the attribute of age-standardization. Whereas the conventional cross-sectional measure is presented in un-standardized terms, the period measure benefits by an implicit age-standardization adjustment (akin to direct age-standardization with a unique reference population as discussed above). The performance of proposed translations from the period to cross-sectional measures were shown to depend rather strongly upon the age-composition of the target populations; with better performance characterizing populations from developed countries that have an advanced average ages (the expected value age of a population computed based on its age composition). Further proof of the influence of the age-standardization attribute on the relationship between cross-sectional and period measures was demonstrated when the the cross-sectional measure was age-standardized. The proposed translation expression in this case, revealed a far tighter and better behaved relationship.

So in summary residual discrepancies in N1 (per-capita) summaries are shown to be traceable to differences in two attributes.

- the “*counting horizon*”
- age-standardization protocols (none versus a type implicit to life-table computations)
- and an adjustment for dyanamic feedback

To account for these dependencies we have incorporated an adjustment for the *counting horizon* attribute and made use of age-standardized counterparts when trying to finesse (adjust for) differences in age-composition across the populations.

We have established that a reasonable proxy for a cross-sectional summary can be obtained by multiplying the traditional life-table measure by the crude mortality rate (ratio of death counts to population counts). The inverse of this relationship suggests that a traditional period measure could be estimated by dividing the applicable cross-sectional measure by the crude-mortality rate (D_T/P_T).

When the cross-sectional measure is itself standardized (employing direct standardization) the proxy for the cross-sectional measure (which involves multiplying the traditional life-table measure by the ASMR) performs far better, aligning more closely with a one-to-one match between exact and proxy values. The residual shortfall in performance is thought to be traceable to differences in the age-composition that are not fully taken into account using the standardization process.

We have defined a taxonomy (See Tables 2 and 4) for defining the distinction between cross-sectional and period based measures. We have demonstrated that these two seemingly distinct approaches can be related to one and another, albeit imprecisely. We have shown that of the distinguishing attributes, the static versus dynamic distinction may be the least influential provided that the scale of excess rate ratio (ε) is moderate to small in size. We have established a rationale for adjusting for the question of ‘what counts’ between the two approaches and have shown the importance of noting differences in two other attributes between the traditional version of the cross-section and period measures. The first of these, normalization, is rather trivial to take into account, whereas the second of these “standardization” requires a bit more more effort.

While the relationship linking the cross-sectional and period measures has been explicated in N1 terms (that is in per capita terms) adjusting to obtain a un-normalized (N0) or a per

attributable death (N2) is straightforward. This has implications for translating between normalization variants of cross-sectional (or period) measures. This shows the ease with which one can move from un-standardized to per-capita or per-death normalized variants. This versatility also enables an analyst to translate between cross-sectional and period measures even when those measures do not start out with a shared normalization variant. So for example the relationships summarized herein would enable an analyst to use a traditional period measure (N1) as a proxy for a traditional cross sectional measure (N0), where these two measures as indicated parenthetically differ in their normalization.

These relationships (tabulated below) may prove more useful, possibly enabling one to scale health impact estimates from one context to another, provided some of the factors identified herein can be safely assumed to be comparatively invariant between the two contexts; in such a case one need only make adjustments for the set of factors known to differentiate the two contexts. For example, one might anticipate that the N2 are relative invariant across countries and across periods (calendar years within a country). If this were the case, perhaps a lions share of scaling an estimate from one context to another can be accomplished by multiplying by a ratio of mortality rates (crude or standardized, depending on whether the measures being scaled are themselves crude or standardized). So it is hoped that the expressions offered herein, which are in the case of period measures approximations, and in the case of cross-sectional measures, exact re-expressions may be of use in enabling wider intuition and insight amongst those pursuing either cross-sectional or period based life-year summaries of health impact.

There are a couple directions that could be pursued to extend this work. There might be some opportunity for improving the performance of the relationships, either by more sophisticated adjustments for the influence of age-composition or more insightful adjustment for the question of what counts. On the question of better finessing the age composition, age standardization, question there may be rather limited gains but they may require a degree of added complexity

that is not warranted given the spirit of this work. We are aiming for parsimonious insight that reveals approximate relationships; intuition.

This analysis has been restricted to quantifying the life-year ramifications of changes in mortality, and has focused on excess rate ratio changes that are exerted on all cause mortality. In future work it might be worth determining relationships that would apply under differing circumstance. First, we would be wise to explore whether similar relationships can be derived for cause-specific changes in mortality, enabling one, for example, to address a situation that imposes an ERR upon lung-cancer mortality or cardiovascular mortality alone. Relationships in these cases ought to be derivable, however, their performance may be subject to other influences above and beyond those that have been examined with respect to the case of all-cause mortality herein. Similarly, an extension to accommodate life-year summary measures that capture both mortality and morbidity impacts may also be of interest. Though here again the performance of the linking relationships would likely be influenced by a wider set of factors (for one thing the age-distribution of morbidity events) than we have noted with respect to the exclusive mortality case. Further, restrictive assumptions regarding the morbidity examined may well need to be invoked in such an analysis. Nonetheless, to the extent that such an analysis would reveal simplifying expressions of the type shared herein; the properties they reveal may be more broadly instructive.

This type of approach of re-expressing calculations of health impact measures in terms of readily identifiable and interpretable baseline summary measures, may be a helpful step towards uncertainty importance operations aimed at identifying the key influential inputs explaining the most variance in outputs. One of the challenges faced by such enterprises in this context of health impact assessment, is the need for age stratification. It yields a large and unwieldy number of inputs (for example mortality rate for each of 22 or 110 age intervals) the influence of any one of which would be less interesting to an analyst than knowing whether a more recognize-

able property of the populations mortality experience was or was not influential in the analysis. The advantage of our re-expression of age-integrating calculations is that it relies on summary measures that natural composites across age, and these composites are readily interpretable, and often more readily available than their constituent parts.

References

- [1] C. J. L. Murray, J. A. Salomon, and C. Mathers. A critical examination of summary measures of population health. In C. J. L. Murray, C. Mathers, J. A. Salomon, and A. D. Lopez, editors, *Summary Measures of Population Health: Concepts, Ethics, Measurement and Applications*, pages 13–40. WHO, Geneva, Switzerland, 2002.
- [2] M. R. Gold, D. Stevenson, and D. G. Fryback. HALYs and QALYs and DALYs, oh my. *Ann. Rev. Public Health*, 23:115–34, 2002.
- [3] I McDowell, RA Spasoff, and B Kristjansson. On the classification of population health measurements. *Am J. Public Health*, 94(3):388–93, 2004.
- [4] Kevin P. Brand. Approximations and Heuristics for the Cause-Modified Life Table. *Risk Analysis: An International Journal*, 25(3):695–709, 2005.
- [5] S. Greenland. Causality theory for policy uses of epidemiological measures. In C. J. L. Murray, C. Mathers, J. A. Salomon, and A. D. Lopez, editors, *Summary Measures of Population Health: Concepts, Ethics, Measurement and Applications*, pages 291–302. WHO, Geneva, Switzerland, 2002.

- [6] J. M. Romeder and J. R. McWhinnie. Potential years of life lost between ages 1 and 70: an indicator of premature mortality for health planning. *International Journal of Epidemiology*, 6(2):143–151, 1977.
- [7] K. Wilkins and E. Mark. Potential Years of Life Lost, Canada, 1990. *Chronic Diseases in Canada*, 13(6):111–115, 1992.
- [8] S. Anand and K. Hanson. Disability-adjusted life years: a critical review. *J. of Health Economics*, 16:685–702, 1997.
- [9] C. D. Mathers, K. M. Iburg, J. A. Salomon, A. Tandon, and et al. Global patterns of healthy life expectancy in the year 2002. *BMC Public Health*, 4(66), 2004.
- [10] Colin Douglas Mathers. Health expectancies : an overview and critical appraisal. 2003.
- [11] R. J. Marshall. Standard Expected Years of Life Lost as a Measure of Disease Burden: An Investigation of Its Presentation, Meaning and Interpretation. In Victor R. Preedy and Ronald R. Watson, editors, *Handbook of Disease Burdens and Quality of Life Measures*, pages 401–413. Springer New York, New York, NY, 2010.
- [12] Roger J. Marshall. Standard expected years of life lost as a measure of mortality: norms and reference to New Zealand data. *Australian and New Zealand Journal of Public Health*, 28(5):452–457, October 2004.
- [13] Christopher J. L. Murray, Colin D. Mathers, Joshua A. Salomon, and Alan D. Lopez. *Chapter 5.1 Health gaps: an overview and critical appraisal*.
- [14] P. Hofstetter and J. K. Hammitt. Selecting human health metrics for environmental decision support. *Risk Analysis*, 22(5):965–984, 2002.

- [15] W. Haenszel. A standardized rate for mortality defined in units of lost years of life. *Amer. J. Public Health*, 40:17–26, 1950.
- [16] R. N. Anderson and H. M. Rosenberg. Age standardization of death rates: implementation of the year 2000 standard. *National Vital Statistics Reports: From the Centers for Disease Control and Prevention, National Center for Health Statistics, National Vital Statistics System*, 47(3):1–16, 20, October 1998.
- [17] Samuel Preston, Patrick Heuveline, and Michel Guillot. *Demography: Measuring and Modeling Population Processes*. Wiley-Blackwell, Malden, MA, October 2000.
- [18] N. Keyfitz. What difference would it make if cancer were eradicated? An examination of the Taeuber paradox. *Demography*, 14(4):411–18, 1977.
- [19] N. Goldman and G. Lord. A new look at entropy and the life-table. *Demography*, 23(2):275–82, 1986.
- [20] J. W. Vaupel. How change in age-specific mortality affects life expectancy. *Population Studies*, 40(1):147–157, 1986.
- [21] Kevin Brand. Health expectancy versus health gap measures: What difference do at-risk dynamics make? In *Proceedings, Society of Risk Analysis (SRA) Conference, Denver, Colorado, December, 2014*.
- [22] Omar B Ahmad, Cynthia Boschi-Pinto, Alan D Lopez, Christopher JL Murray, Rafael Lozano, and Mie Inoue. Age Standardization of Rates: A new WHO Standard. page 14, 2001.

Description of term	Cross-sectional	Period
Number of deaths in the interval $[x, x + 1)$	D_x	d_x
Population count within the interval $[x, x + 1)$	P_x	L_x
Mortality rate (deaths/person-time) within interval $[x, x + 1)$	M_x	M_x
Proportion alive at age x		$S_x (= l_x/l_0)$
Residual life-expectancy at age x (period LT calculation)	e_x^o	e_x^o
Total number of deaths	D_T	l_0
Total population	P_T	l_0
Average (across age) mortality rate	CMR	$1/e_0^o$
Age standardized mortality rate	ASMR	$1/e_0^o$

Table 5: Contrasts the nomenclature used herein to refer to cross-sectional constructs (used in calculating Health Gaps) versus the nomenclature used to refer to *period* life-table constructs (used in calculating Health Expectancies). The nomenclature D_x connotes a count (for deaths) that occurred within an age-interval extending from age x to $x + 1$. In this paper we follow a convention of dropping the pre-subscript n noting that all age-intervals are a single year in size.

[23] Kevin Brand and David Stieb. Quantifying health impacts in life-years: A bridge between two methods. In *Proceedings, Society of Risk Analysis (SRA) Conference, Charleston, South Carolina, December, 2011*.

Comparing Concepts/notation

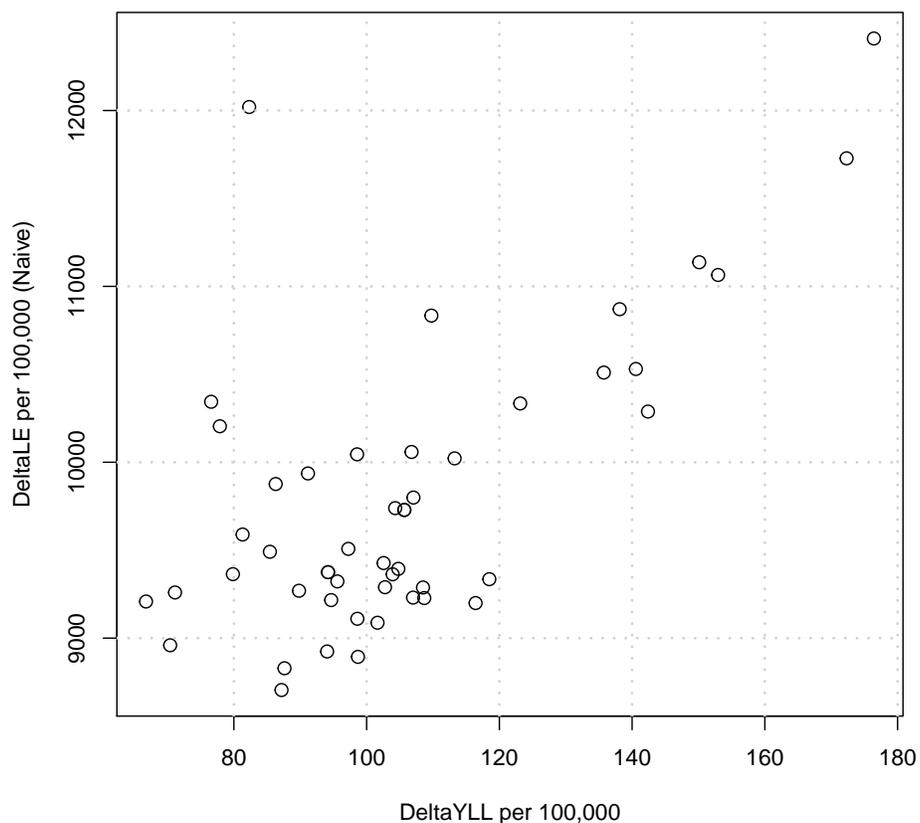


Figure 3: Estimates of ΔYLL (horizontal axis) are plotted against their *naive* ΔLE counterparts for 49 countries — so-called *naive* because no effort has been made to adjust for what we refer to in the text as the *counting horizon*. Estimates are based on female counts, rates, and life-tables for the period of 2005. The ΔYLL measure is an N1 variant (reflecting a per-capita measure) while its ΔLE counterpart is understood in per capita terms as well. The paired summaries are evidently on different scales — the period (ΔLE) results are roughly 100-fold higher than their cross-sectional counterparts, though the pattern of correlation is not strong. Results are expressed in years per 100,000 people.

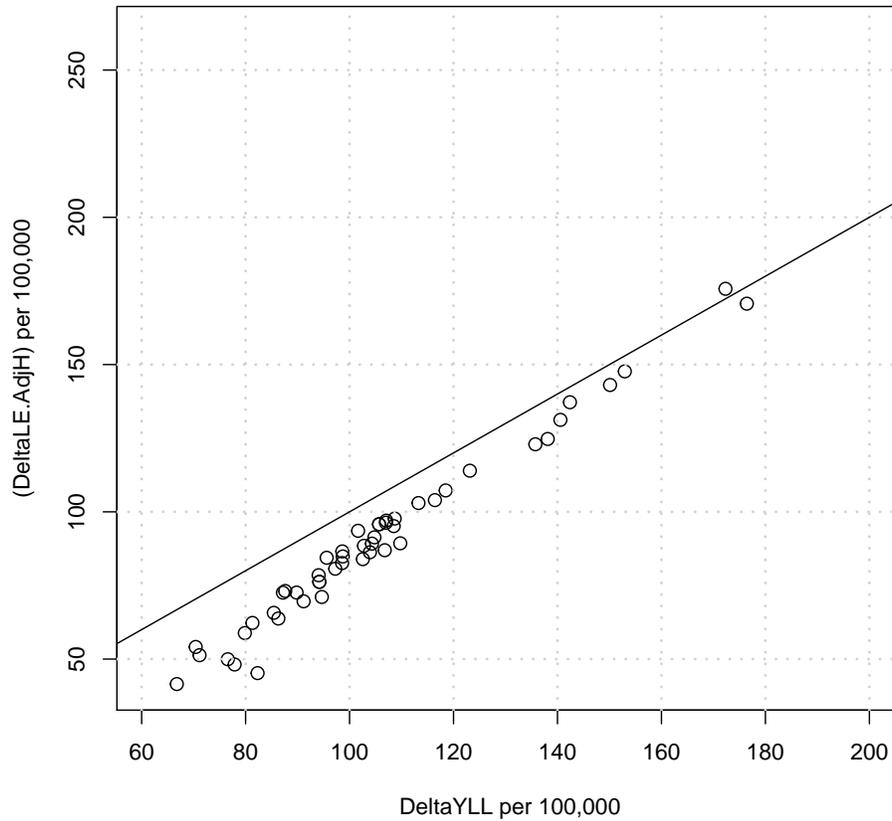


Figure 4: Estimates of ΔYLL (horizontal axis) are plotted against their *adjusted* ΔLE counterparts, where *adjustment* refers to adjusting for the *counting horizon*. Estimates are based on female counts, rates, and life-tables for 49 countries, and for the period of 2005 (data source is the HMD). Results are expressed in years per 100,000 people.

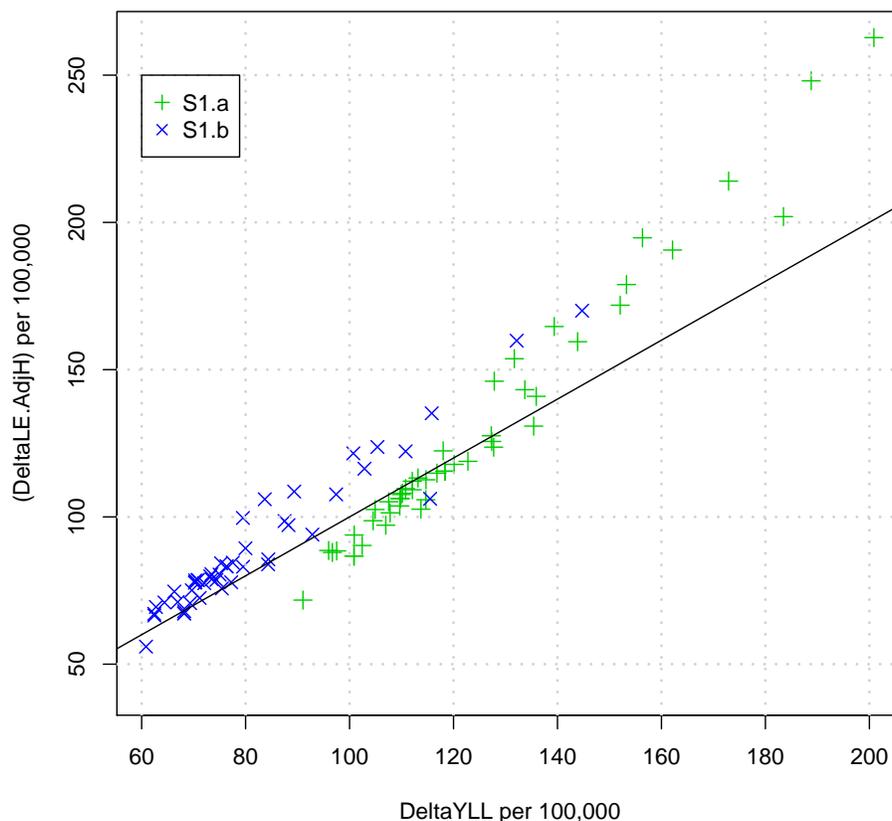


Figure 5: Age-standardized (S1) estimates of ΔYLL (horizontal axis) are plotted against their *adjusted* ΔLE counterparts, where *adjustment* refers to adjusting for the *counting horizon*. This adjustment is different under S1 case than the S0 case that was the subject of previous scatterplots. Directly standardized HI series require scaling by the appropriate ASMR rather than the CMR (as was done in the S0 case). Two matched data series are shown one (S1a) using the average L_x schedule as the referent population and the other (S1b) using the WHO referent population. Estimates are based on female counts, rates, and life-tables for 49 countries, and for the period of 2005 (data source is the HMD). Results are expressed in years per 100,000 people.

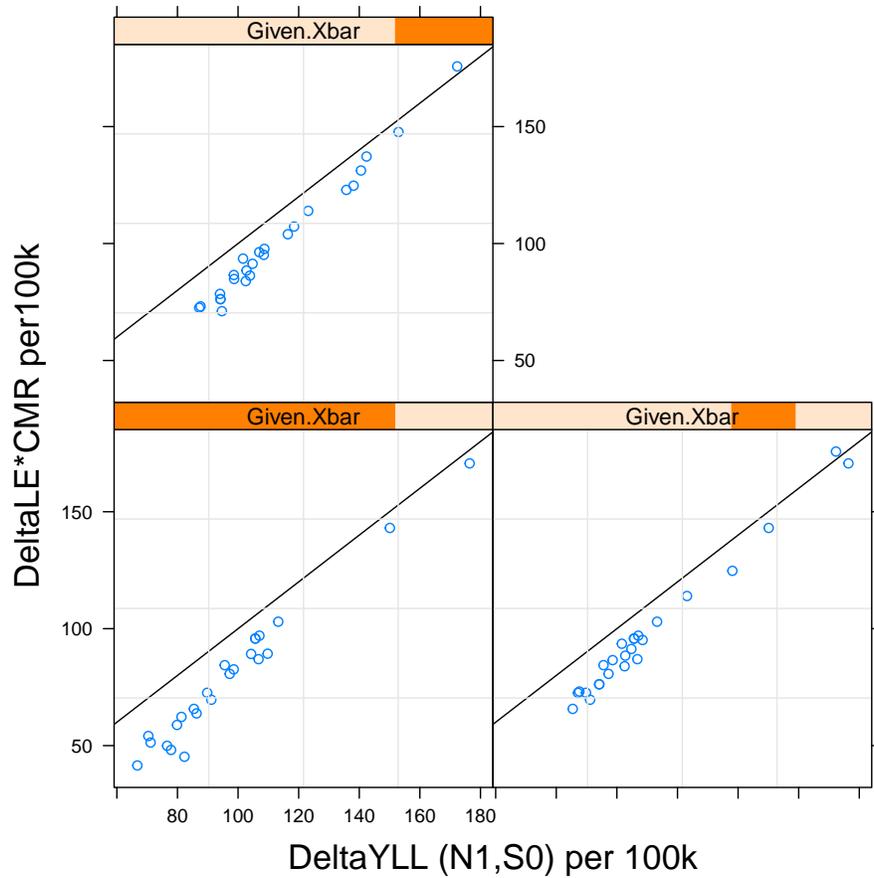


Figure 6: Revisits Fig 4 separating the matched data-series into three equal and overlapping subgroups, ordered from smallest (left panel) to largest (rightmost panel) average age of country's underlying age-distribution. Results are expressed in years per 100,000 people.

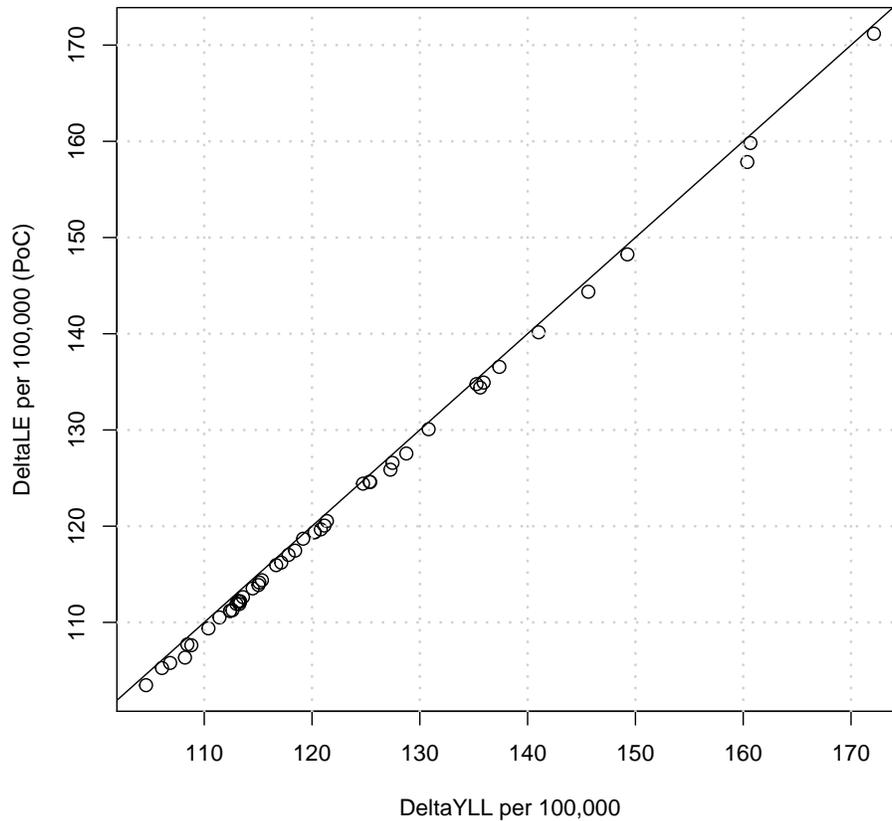


Figure 7: Scatterplot tests our *proof of concept*, namely whether an identity might be formed between ΔYLL measures and their period, ΔLE , counterparts. As indicated in the methods, using a country-specific referent, namely its L_x schedule (from its life-table) in a direct age-standardization protocol is predicted to achieve near identity. Results are expressed in years per 100,000 people.

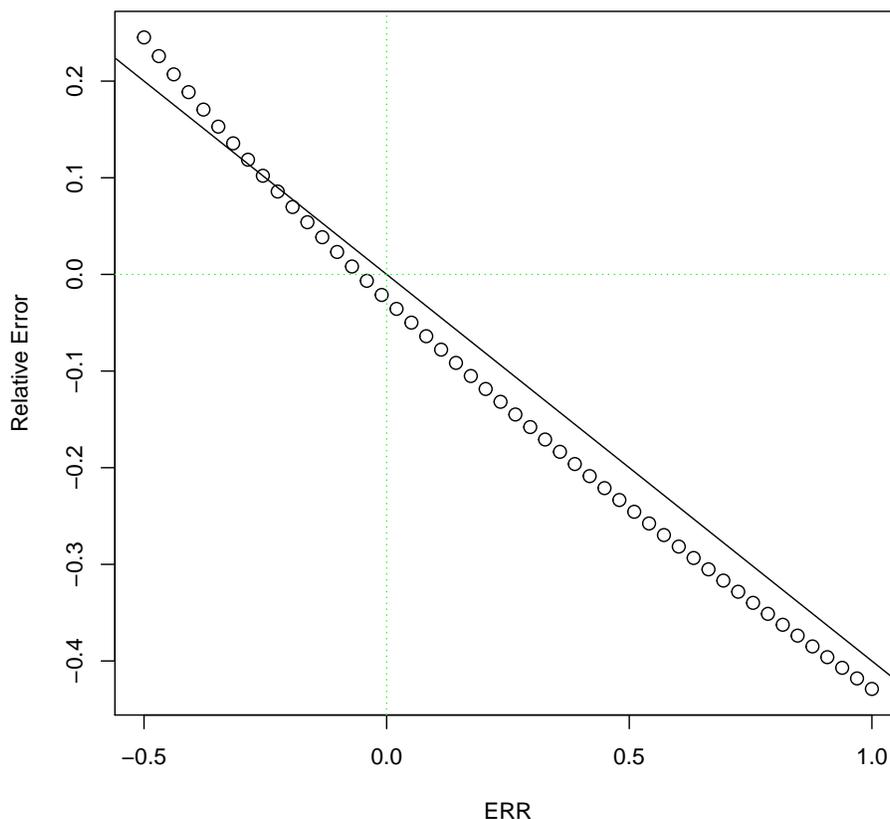


Figure 8: Plots the relative error (RE) between the *Exact* [?] and *Approximate* [18, 4] expressions for *DLE* as a function of a sequence of excess rate ratios (ERR or equivalently ε). Here $RE = (Exact - Approx)/Exact$. In previous work [21] one of us proposed a simple formula for this relative error that was shown to perform well for developed countries among a set of 194 WHO country life-tables. Results are based on Canadian, female, life-tables for the period 2005, and like all results herein present the case for “all-cause” mortality perturbations. Cause-specific perturbations entail a different locus for RE and a different adjustment $(AF)_{dyn}$. Applying the previously fitted relationship to this 2005 data reveals a reasonable match. This same result informs our specification of an adjustment $(AF)_{dyn}$ for dynamics that can be applied to cross-sectional results so that they capture the feedback that would apply, especially if the estimate is conceptualized as applicable to a long sequence of successive years.