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economic prosperity: semi-endogenous growth
revisited

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Public education, technological change and economic prosperity: semi-endogenous growth revisited

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Abstract

We introduce publicly funded education into R&D based economic growth theory. Our framework allows us to i) explicitly describe a realistic process of human capital accumulation within these types of growth models, ii) reconcile semi-endogenous growth theory with the empirical evidence on the relationship between economic development and population growth, iii) revise the policy invariance result of semi-endogenous growth frameworks. In particular, we show that the model supports a negative (positive) association between economic growth and population growth if the education sector is well (badly) developed and that changes of public investments into education crucially affect the long-run balanced growth path.

JEL classification: I25, J24, O11, O31, O41

Keywords: public education, human capital accumulation, technological change, semi-endogenous economic growth

1 Introduction

Over the last decades the roles of human capital accumulation and education in the process of economic development has been analyzed extensively. Most empirical studies find a positive association between economic growth and measures for educational attainment (see for example Mankiw et al., 1992; Barro and Lee, 1994; Sachs and Warner, 1995; Hall and Jones, 1999; Bils and Klenow, 2000)¹ and Lutz et al. (2008) conclude that

“...better education does not only lead to higher individual income but also is a necessary (although not always sufficient) precondition for long-term economic growth... Education is a long-term investment associated with near-term costs, but, in the long run, it is one of the best investments societies can make in their futures.” (Lutz et al., 2008, p. 1048).

Despite the seminal theoretical contributions of Lucas (1988), Galor and Weil (2000) and Galor (2005) highlighting the mechanisms by which education and human capital accumulation exert their influence on economic development, the main focus of endogenous and semi-endogenous growth theory has long been on technological progress shown to be determined by the research and development (R&D) effort of an *uneducated* workforce. In one of the first models of this type, Romer (1990) acknowledges that human capital and not raw labor is what matters but this notion is not explicitly modeled. To put it differently, within these types of models, the aggregate human capital stock exhibits the same behavior as raw labor and therefore issues of human capital accumulation and education cannot be addressed. However, to underscore the vast importance of changes in education over the last decades, table 1 shows mean years of schooling of the population aged 15+ for the years 1960 and 2010 in the G-8 countries. Basically, we see that there has been a huge increase over time, with annual growth rates in between 0.5% and 2%.

Early R&D based endogenous growth models in the vein of Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) furthermore supported a strong scale effect in the sense that the size of a countries' population determined its long-run economic growth prospects. The intuitive explanation was that larger populations feature i) larger markets and

¹However, the significance of this association and the direction of causality are often debated (cf. Durlauf et al., 2005).

Table 1: Mean years of schooling for the G-8 countries 1960 versus 2010 obtained from Barro and Lee (2010)

Country	1960	2010
Canada	8.31	11.37
France	4.20	10.53
Germany	5.15	11.82
Italy	4.86	9.88
Japan	8.02	11.59
Russia	5.16	8.84
U.K.	7.04	9.75
USA	9.25	12.20

therefore higher profit opportunities for innovative firms introducing new products, and ii) a larger pool of human capital, i.e., more potential researchers available for R&D to propel technological progress. While it has been shown by Kremer (1993) that the scale effect was indeed important in economic history prior to the twentieth century for the world as a whole, it has been refuted for example by Jones (1995a) and Jones (1995b) for individual countries and their growth experiences in the second half of the twentieth century. This paved the way for semi-endogenous growth models (cf. Jones, 1995a; Kortum, 1997; Segerström, 1998), which remove the strong scale effect in a way that the long-run economic growth rate positively depends on population *growth* but not its *size*. The basic intuitive argument runs as follows: keeping up technological progress at an exponential rate becomes more and more difficult as the technological frontier expands. Therefore, a constant inflow of scientists into the R&D sector is required to counterbalance this effect. In the long run, such a constant inflow can only be insured by having positive population growth. However, even this implication has been severely criticized on the basis of empirical investigations which rather support a *negative* association between economic growth and population growth (see for example Brander and Dowrick, 1994; Kelley and Schmidt, 1995; Ahituv, 2001; Bernanke and Gürkaynak, 2001). Furthermore, the removal of the strong scale effect came at the price that the long-run economic growth rate within this model class was invariant to economic policy and solely depended on the exogenously given population

growth rate and the extent of intertemporal knowledge spillovers. A lot of research has been carried out to reintroduce a role for economic policy in scale-free economic growth frameworks (cf. Dinopoulos and Thompson, 1998; Peretto, 1998; Young, 1998; Howitt, 1999) but the resulting models were also subject to the critique of suggesting a positive relationship between economic growth and population growth in the long run.

Some recent attempts have been made to reconcile theory and evidence by Dalgaard and Kreiner (2001), Strulik (2005) and Strulik et al. (2011) who show that it is not the sheer size of the labor force but also its education that matters. In addressing this issue they implement privately financed education into R&D based growth models. While Dalgaard and Kreiner (2001) and Strulik (2005) emphasize that newborns do not have any education and therefore a larger birth rate essentially slows down growth of average human capital, Strulik et al. (2011) introduce a child quality-quantity trade-off following Becker (1993) and show that a shift towards having fewer but better educated children leads to a larger aggregate human capital stock within an economy and therefore to faster economic growth. This implication in turn has been challenged by Prettnner et al. (2012), who argue for a more general description of the education sector and do not find empirical evidence that the quality-quantity trade-off is — as an isolated mechanism — strong enough to overturn the negative influence of declining fertility on aggregate human capital.

The aim of our paper is therefore to achieve three goals. First, we want to implement a realistic description of *publicly* financed education into R&D based economic growth models and thereby contribute to bridging the gap between technological progress and human capital accumulation in the theoretical economic growth literature. While the assumption of *privately* financed education could be justified for the United States, it might not fit for European countries, where educational systems are often entirely financed by the state. Second, we aim to reconcile theory and evidence by showing that our framework allows for both a negative and a positive relationship between economic growth and population growth, where the negative relationship is more likely to prevail for countries in which the education sector is well developed, i.e., typically for modern industrialized countries. Third, we attempt to reintroduce scope for policymakers to influence the long-run economic growth rate and show that public expenditures for education are

crucial in this regard. This not only addresses a major concern expressed by the proponents of scale-free economic growth models with scope for policymakers to intervene, but it is also consistent with the vast empirical literature on the interrelationship between education and economic prosperity.

The paper proceeds as follows: section 2 contains the theoretical model that allows us to analytically derive the dynamical system fully describing our model economy and to determine the growth rates of the crucial variables along the balanced growth path. Furthermore, we analytically assess the dependence of these growth rates on the underlying parameters, in particular, population growth and public education expenditures. In section 3 we numerically analyze the implications of an increase in public educational expenditures for economic growth during the transitions to the new balanced growth path. Finally, section 4 discusses the results, draws conclusions for economic policy and highlights scope for further research.

2 The model

This section describes the discrete time overlapping generations version of the R&D based economic growth framework based upon Romer (1990) and Jones (1995a). Furthermore, we introduce a governmentally funded education sector and analyze its implications for long-run economic growth perspectives.

2.1 Basic assumptions

The demographic structure of our model economy follows Diamond (1965) and is a simplified version of Strulik et al. (2011). There are three phases of an individual's life cycle, each lasting for 25 years: childhood, adulthood and retirement. Children do not face economic decisions but they receive publicly funded education which determines their human capital level as an adult. Adults, whose cohort size at time t is given by L_t , inelastically supply their skills on the labor market, consume and save for retirement. The retirees in turn finance their consumption expenditures out of savings carried over from adulthood. For expositional reasons, we treat population growth as exogenous and assume that adults give birth to $n > 1$ children

such that the population grows at rate $n - 1$.²

There are four sectors: final goods production, intermediate goods production, R&D and education. Two production factors can be used in these sectors: capital and labor. The latter is available in three different forms: i) workers in the final goods sector, denoted by $L_{t,Y}$, ii) scientists in the R&D sector, denoted by $L_{t,A}$, and iii) teachers in the education sector, denoted by $L_{t,E}$. The final goods sector employs workers and machines supplied by the intermediate goods sector to produce for a perfectly competitive consumption good market. The Dixit and Stiglitz (1977) monopolistically competitive intermediate goods sector produces the machines for the final goods sector using capital as variable production factor and one machine specific blueprint as fixed input. This blueprint is in turn supplied by the R&D sector which employs scientists to produce them. Finally, the education sector employs teachers to produce individual human capital for the next generation, denoted by h_{t+1} . Following Mankiw et al. (1992) by assuming that human capital and raw labor are perfect substitutes allows us to write aggregate human capital employment as $H_t = L_t h_t$. Furthermore, expenditures for the education sector are financed by taxing wages of adult workers.

2.2 Consumption side

Suppose that adults maximize their discounted lifetime utility determined by consumption in adulthood and after retirement in the vein of Diamond (1965)

$$\max_{c_t, s_t} u_t = \log c_t + \beta \log(r_{t+1} s_t), \quad (1)$$

where c_t denotes consumption, s_t represents savings carried over to retirement, $\beta = 1/(1 + \rho)$ refers to the discount factor with ρ being the discount rate and r_{t+1} denotes the net interest rate paid on assets between generation t and generation $t + 1$. Note that each time period corresponds to one generation and therefore lasts for 25 years. Assuming full depreciation of capital over the course of one generation, the gross interest rate is given by

²Endogenizing population growth and private educational investments would allow us to analyze interrelations between private and public education. This is suggested for further research because it would severely complicate the model structure and obscure the basic mechanisms we aim to highlight. See also the discussion in section 4.

$R_{t+1} = r_{t+1} + 1$. The budget constraint of a young adult reads

$$(1 - \tau)w_t h_t + l_t = c_t + s_t, \quad (2)$$

where τ denotes the income tax rate, w_t represents the wage per efficiency unit of labor and l_t are lump-sum redistributions of the monopolistic rents siphoned by the government after a patent has expired (see section 2.3.3 for details). The results of the maximization problem are expressions for optimal consumption and savings

$$c_t = \frac{l_t + (1 - \tau)h_t w_t}{1 + \beta}, \quad (3)$$

$$s_t = \frac{\beta (l_t + (1 - \tau)h_t w_t)}{1 + \beta}, \quad (4)$$

exhibiting the standard properties for logarithmic utility, i.e., they are increasing in wage income and lump-sum governmental transfers and decreasing in tax rates and the discount factor because the latter reduces savings and thereby lifetime interest income.

2.3 Production side

This subsection describes the production structure in the four sectors final goods production, intermediate goods production, R&D and education. The treatment of the former two sectors is fairly standard (cf. Romer, 1990; Jones, 1995a; Strulik et al., 2011) and can be brief. However, we augment these frameworks to account for an income tax financed public education sector that employs labor to produce human capital of individuals and thereby increases the productivity of subsequent generations. Consequently, the education sector competes with the R&D sector and with the final goods sector on the labor market.

2.3.1 Final goods sector

Final output Y_t being consumed by the adults and retirees in the economy and representing the the gross domestic product (GDP) is produced accord-

ing to the production function

$$Y_t = H_{t,Y}^{1-\alpha} \sum_{i=1}^{A_t} x_{t,i}^\alpha, \quad (5)$$

where $H_{t,Y}$ is human capital employed in the final goods sector, A_t is the technological frontier, i.e., it represents the most modern blueprint that has been developed in the R&D sector, $x_{t,i}$ is the amount of the blueprint-specific machine i used in final goods production and α is the intermediate share of final output. Due to perfect competition in the final goods market, production factors are paid their marginal products such that the wage rate per unit of human capital and prices of blueprints are determined as

$$w_{t,Y} = (1 - \alpha) H_{t,Y}^{-\alpha} \sum_{i=1}^{A_t} x_{t,i}^\alpha = (1 - \alpha) \frac{Y_t}{H_{t,Y}}, \quad (6)$$

$$p_{t,i} = \alpha H_{t,Y}^{1-\alpha} x_{t,i}^{\alpha-1}. \quad (7)$$

Note that the derived prices for machines rely on the property that individual intermediate goods producing firms are deemed to be small in comparison to the whole sector. Consequently, the contribution of one such firm to the output of the whole sector can be neglected.³

2.3.2 Intermediate goods sector

We assume that a single intermediate goods producer is able to convert capital $k_{t,i}$ one for one into machines $x_{t,i}$ after it has purchased the corresponding blueprint from the R&D sector. Therefore its operating profits read

$$\pi_{t,i} = p_{t,i} k_{t,i} - R_t k_{t,i} \quad (8)$$

and profit maximization leads to the familiar outcome of Dixit and Stiglitz (1977) that firms charge prices for machines that are a markup $1/\alpha$ over marginal cost. Therefore we have

$$p_{t,i} = \frac{R_t}{\alpha} \quad (9)$$

³Sometimes an integral is used instead of the sum to address this issue.

and we see that there is symmetry between firms such that the index i can be dropped. As another consequence of symmetry, we know that each firm employs $k_t = K_t/A_t$ units of capital, where we denote the aggregate capital stock by K_t . Consequently, we can rewrite the aggregate production function as

$$Y_t = (A_t H_{t,Y})^{1-\alpha} K_t^\alpha, \quad (10)$$

where we see that technology is human capital augmenting.

2.3.3 R&D sector

The R&D sector employs scientists with a human capital level $H_{t,A}$ and productivity δ in order to develop new blueprints. Therefore the production function of a firm in the research sector can be written as

$$A_{t+1} - A_t = \delta A_t^\phi H_{t,A}, \quad (11)$$

where ϕ measures the extent of intertemporal knowledge spillovers. In case that $\phi = 1$ we would be in the Romer (1990) environment and sustaining an exponential growth rate of technology does not become ever more complex as the technological frontier expands. We see from equation (11) that a constant amount of human capital in research would then suffice to have technological progress indefinitely and therefore economic growth even in the long run. By contrast, if $\phi < 1$, we are in the Jones (1995a) environment and a constant growth rate of technology either requires a constant inflow of additional scientists into R&D, or a continuous increase in education of the scientists already employed, or both. Since we have positive population growth and human capital accumulation, no balanced growth path would exist in the Romer (1990) environment such that we assume $\phi < 1$ to hold from now on. Firms in the R&D sector maximize their profits

$$\pi_{t,A} = p_{t,A} \delta A_t^\phi H_{t,A} - w_{t,A} H_{t,A} \quad (12)$$

with $p_{t,A}$ being the price of a blueprint and $w_{t,A}$ being wages of scientists. This leads to the optimality condition

$$w_{t,A} = p_{t,A} \delta A_t^\phi, \quad (13)$$

where we see that wages of scientists increase in prices for blueprints.

It is assumed that patent protection for a newly discovered blueprint lasts for one generation. Afterwards the right to sell the blueprint is handed over to the government which redistributes the proceeds in a lump-sum manner. This assumption simplifies the exposition considerably and allows us tracing the transitional dynamics because in contrast to standard endogenous and semi-endogenous growth models, we do not require interest rates to remain constant over time (cf. Strulik et al., 2011, for a comparable mechanism). Therefore R&D firms can charge prices for blueprints that are equal to the operating profits of intermediate goods producers in time period t (when patent protection is valid) because there is always a potential entrant willing to pay that price. To put it differently, in case that blueprints were less (more) expensive, firms would have an incentive to enter (exit) the market. Consequently, we can write

$$p_{t,A} = (\alpha - \alpha^2) \frac{Y_t}{A_t} \quad (14)$$

which follows from equations (7) and (9) and the fact that $x_i = k_i$ for all i .

2.3.4 Education sector

Finally, we have the education sector that employs teachers financed by the proceeds of income taxes in order to produce human capital.⁴ We assume a balanced governmental budget such that we have

$$\tau w_t h_t L_t = w_t h_t L_{t,E}, \quad (15)$$

where the left hand side represents governmental revenues, i.e., the proceeds of taxing the total wage bill $w_t h_t L_t$, and the right hand side represents governmental expenditures, i.e., the wage bill paid for teachers. This implies that the number of employed teachers is $L_{t,E} = \tau L_t$. Next, we assume that the education sector produces effective years of schooling, denoted by ys_t , according to

$$ys_t = \xi \frac{L_{t,E}}{n L_t} = \xi \frac{\tau}{n}, \quad (16)$$

⁴Gersbach et al. (2009) use a comparable financing scheme for basic research in a hierarchical growth model.

where ξ measures the productivity of teachers and τ/n denotes the teacher-pupil ratio. This implies that effective years of schooling increase in the productivity of teachers and in public educational investments per child. Building upon Mincer (1974) and following Hall and Jones (1999), Bils and Klenow (2000) and Caselli (2005), effective years of schooling translate into individual human capital according to

$$h_{t+1} = \exp \left[\psi \left(\xi \frac{\tau}{n} \right) \right] h_t, \quad (17)$$

where $\psi(\cdot)$ measures the extent to which they do. For example, Bloom and Canning (2005) use a linear relationship with a constant of 0.091 for years of education based upon evidence by Psacharopoulos (1994), while Hall and Jones (1999) assume a piecewise linear function that takes a value of 0.134 for years of primary, 0.101 for years of secondary and 0.068 for years of tertiary education. To keep things simple, we adopt the former approach of a linear relationship. Altogether, equation (17) implies that if the government does not invest into education at all, human capital of the successive generation will be the same as those of their parents. This can at least partly be justified by the notion that without formal education, people are observing and learning from their parents and peers (cf. Strulik et al., 2011, p. 8). Furthermore, the model would lack positive economic growth in the balanced growth path without this implication which would be at odds with stylized facts of development in modern economies (cf. Acemoglu, 2009; Galor, 2011).⁵

2.4 Market clearing and the balanced growth path of the economy

Labor market clearing implies that the total amount of available human capital has either to be employed in the final goods sector, in the education sector or in the R&D sector, i.e., we have that $h_t L_t = h_t (L_{t,E} + L_{t,A} + L_{t,Y}) \Rightarrow H_t = H_{t,E} + H_{t,A} + H_{t,Y}$. Furthermore, we know that wages in all sectors have to equalize such that $w_{t,E} = w_{t,A} = w_{t,Y}$, otherwise one or more sectors

⁵Of course it can be questioned whether a positive economic growth rate can be sustained indefinitely facing scarce resources, a limited carrying capacity of the environment and bounded space on earth. However, we want to emphasize that we do not insist that our model holds for $t \rightarrow \infty$ but that it represents a reasonable approximation for a certain period of time.

would not be able to attract any workers and the economy would end up in a corner solution. Equalizing expressions (6) and (13), using equation (14) and noting that employment in the education sector is τL_t , yields demand for workers in the final goods sector and in the R&D sector as

$$H_{t,Y} = \frac{A_t^{1-\phi}}{\alpha\delta}, \quad (18)$$

$$H_{t,A} = (1 - \tau)H_t - \frac{A_t^{1-\phi}}{\alpha\delta}. \quad (19)$$

We see that an increase in the population size or in individual human capital leads to more employment of aggregate human capital in education and in science. The latter fosters faster technological progress such that A_{t+1} rises by more than it would have otherwise. This in turn increases human capital employment in the final goods sector in generation $t + 1$. Altogether the development of new blueprints can be described by

$$A_{t+1} = \delta(1 - \tau)A_t^\phi h_t L_t - \frac{1 - \alpha}{\alpha} A_t, \quad (20)$$

where we see the basic trade-off that public educational investments imply: While increasing taxes poaches labor from the R&D sector to the education sector, it also increases human capital accumulation and therefore the productivity of the next generation's scientists.

Full depreciation of capital and capital market clearing imply that the aggregate capital stock of an economy in generation $t+1$ is equal to aggregate savings. Furthermore, goods market clearing ensures aggregate consumption together with aggregate savings are equal to total output such that we have

$$K_{t+1} = s_t L_t = Y_t - c_t L_t. \quad (21)$$

These identities can then be used to eliminate the lump-sum redistributions of the government to the households. After doing so, we can derive the equation governing the accumulation of aggregate capital as

$$K_{t+1} = \frac{\beta}{1 + \beta} \left(\frac{A_t^{2-\phi}}{\alpha\delta} \right)^{1-\alpha} K_t^\alpha. \quad (22)$$

Putting all information together, the system fully describing the equilibrium

dynamics of our model economy reads

$$A_{t+1} = \delta(1 - \tau)A_t^\phi h_t L_t - \frac{1 - \alpha}{\alpha} A_t, \quad (23)$$

$$h_{t+1} = \exp\left(\frac{\psi\xi\tau}{n}\right) h_t, \quad (24)$$

$$L_{t+1} = nL_t, \quad (25)$$

$$K_{t+1} = \frac{\beta}{1 + \beta} \left(\frac{A_t^{2-\phi}}{\alpha\delta}\right)^{1-\alpha} K_t^\alpha. \quad (26)$$

Note that these equations hold during the transition to the long-run balanced growth path as well as along the long-run balanced growth path itself. Making use of the definition of a balanced growth path, i.e., that the growth rate of a variable does not change over time, we can derive the rate of technological progress as

$$g_A = [(g_h + 1)(g_L + 1)]^{\frac{1}{1-\phi}} - 1 = \left[\exp\left(\frac{\psi\xi\tau}{n}\right) n\right]^{\frac{1}{1-\phi}} - 1, \quad (27)$$

where g_j denotes the growth rate of variable j . For the aggregate capital stock it follows either from equation (26) or from inspection of the aggregate production function that its long-run balanced accumulation rate is given by

$$\begin{aligned} g_K &= (g_h + 1)(g_L + 1)(g_A + 1) - 1 = \left[\exp\left(\frac{\psi\xi\tau}{n}\right) n\right]^{\frac{2-\phi}{1-\phi}} - 1 \\ &= (g_A + 1)^{2-\phi} - 1. \end{aligned} \quad (28)$$

Denoting per capita GDP by y_t and putting things together, we therefore know that the growth rates of aggregate GDP and per capita GDP are

$$g_Y = (g_h + 1)(g_L + 1)(g_A + 1) - 1 = \left[\exp\left(\frac{\psi\xi\tau}{n}\right) n\right]^{\frac{2-\phi}{1-\phi}} - 1, \quad (29)$$

$$g_y = (g_h + 1)(g_A + 1) - 1 = \left[\exp\left(\frac{\psi\xi\tau}{n}\right)\right]^{\frac{2}{1-\phi}} n^{\frac{1}{1-\phi}} - 1. \quad (30)$$

We see that technological progress is driven by growth in aggregate human capital which is composed of individual human capital and the population size. It might seem that a decrease in both of these variables decrease the

long-run growth rate of the economy. This, however, misses the point that human capital accumulation is inversely related to the population growth rate via the latter's negative influence on the teacher-pupil ratio. The question which of the two effects prevails will be discussed in proposition 1.

We also see that per capita GDP, the crucial measure for prosperity in growth theory, not only increases with the rate of technological progress but also with the rate of individual human capital accumulation. The whole process is then complemented by physical capital accumulation in order to ensure a constant capital-labor ratio and positive growth of per capita GDP even in the long run. Therefore the balanced growth path of the model is consistent with the stylized facts of economic development expressed by Kaldor (1957). Now we can state the first central analytical result of our paper.

Proposition 1. *The long-run growth rates of technology and per capita GDP decrease in response to faster population growth if the education sector of an economy is well-developed. The converse holds true for a badly developed education sector.*

Proof. We take the derivatives of the growth rate of technology and per capita GDP which read

$$\frac{\partial g_A}{\partial n} = \frac{\left[\exp\left(\frac{\psi\xi\tau}{n}\right) n \right]^{\frac{1}{1-\phi}} (n - \xi\tau\psi)}{n^2(1 - \phi)}, \quad (31)$$

$$\frac{\partial g_y}{\partial n} = \frac{\left[\exp\left(\frac{\psi\xi\tau}{n}\right) n \right]^{\frac{2-\phi}{1-\phi}} (n - \xi\tau(2 - \phi)\psi)}{n^3(1 - \phi)} \quad (32)$$

The first expression is negative if the state of the education sector — as measured by the product of public investments into education represented by taxes, τ , productivity of teachers, ξ , and the Mincerian coefficient measuring the translation of effective years of schooling into human capital, ψ , — is very good. Qualitatively the same result holds true for the growth rate of per capita GDP. \square

The economic intuition behind these results is that growth of aggregate human capital is either due to individual human capital or due to growth of the population size. An increase in population growth, which — by

itself — positively impacts upon aggregate human capital accumulation, simultaneously increases the pupil-teacher ratio. This in turn has a negative impact on the evolution of aggregate human capital. If the education sector is well developed, the negative effect will dominate. This is most likely to be the case for developed countries which would be consistent with the evidence found by Brander and Dowrick (1994), Kelley and Schmidt (1995), Ahituv (2001) and Bernanke and Gürkaynak (2001). If, on the other hand, the education sector is badly developed, the other effect will dominate and population growth is positively associated with economic growth. This is most likely to be the case for countries in an early stage of development which would be consistent with the evidence found by Kremer (1993).

Another interesting aspect is that the proof of proposition 1 indicates that there exists a parameter range for which technological progress negatively depends on an increase in population growth, while the converse holds true for per capita output growth. The reason is that the effectiveness of the education sector is multiplied by $2 - \phi > 1$ in the derivative of g_y with respect to n . The intuitive explanation is that individual human capital accumulation not only exerts its positive growth effect via the R&D sector but additionally works along the channel suggested by Lucas (1988), i.e., it increases productivity of workers in the final goods sector. Since we know that a faster accumulation of human capital of workers is accompanied by faster physical capital accumulation, constant returns with respect to these two production factors in aggregate production imply an additional positive impact of education on output growth. Now we turn to the second central analytical result of our paper.

Proposition 2. *The long-run growth rates of technology and per capita GDP unambiguously increase if public investments into education are raised.*

Proof. We take the derivatives of the growth rate of technology and per capita GDP which read

$$\frac{\partial g_A}{\partial \tau} = \frac{\left[\exp\left(\frac{\psi \xi \tau}{n}\right) n \right]^{\frac{1}{1-\phi}} \xi \psi}{n(1-\phi)}, \quad (33)$$

$$\frac{\partial g_y}{\partial \tau} = \frac{\left[\exp\left(\frac{\psi \xi \tau}{n}\right) n \right]^{\frac{2-\phi}{1-\phi}} \xi (2-\phi) \psi}{n^2(1-\phi)} \quad (34)$$

Since both of them are positive, the proposition holds. \square

This result is different from standard semi-endogenous growth models (cf. Jones, 1995a; Kortum, 1997; Segerström, 1998) because it suggests scope for economic policy to influence the long-run economic growth rate which would rather be in line with the second wave of scale-free economic growth models (cf. Dinopoulos and Thompson, 1998; Peretto, 1998; Young, 1998; Howitt, 1999; Dalgaard and Kreiner, 2001). The policy measure to be taken is to increase investments into public education. In this regard our model is also consistent with the literature suggesting a positive association between education and economic growth (cf. Barro and Lee, 1994; Sachs and Warner, 1995; Hall and Jones, 1999; Bils and Klenow, 2000; Lutz et al., 2008). The reason for this effect to prevail is that in the long-run and for a constant population growth rate n , there is only a positive effect of increasing education on aggregate human capital accumulation and hence effective labor unambiguously increases in all sectors of the economy. However, in the medium run, i.e., during the transition to the new balanced growth path, there could also be negative growth effects of increases in public educational investments because the education sector draws labor from the R&D sector. The consequence is that the “near term costs” Lutz et al. (2008) mention could be very pronounced. This is the subject of the next section, where we simulate an increase in educational expenditures and therefore keep track of the medium-term costs as well as of the long-term benefits.

3 Simulating an increase in public educational expenditures

To address the question how the model economy is affected by an increase in public educational expenditures in the medium run as well as in the long run, we simulate the dynamic system displayed in equations (23) to (26) in the software package developed by Diks et al. (2008). The parameter values and justifications for using them are given in table 2. We try to choose parameters to be consistent with data on the growth process of the United States obtained from World Bank (2012) or otherwise to be in line with the corresponding literature. Nevertheless, we emphasize that we are not attempting to calibrate our model to fit historic data for a specific

country which would be futile having time intervals of 25 years and a model that abstracts from many important aspects of reality. However, we aim to present a reasonably justified picture of the medium-run response to an increase in public educational investments.

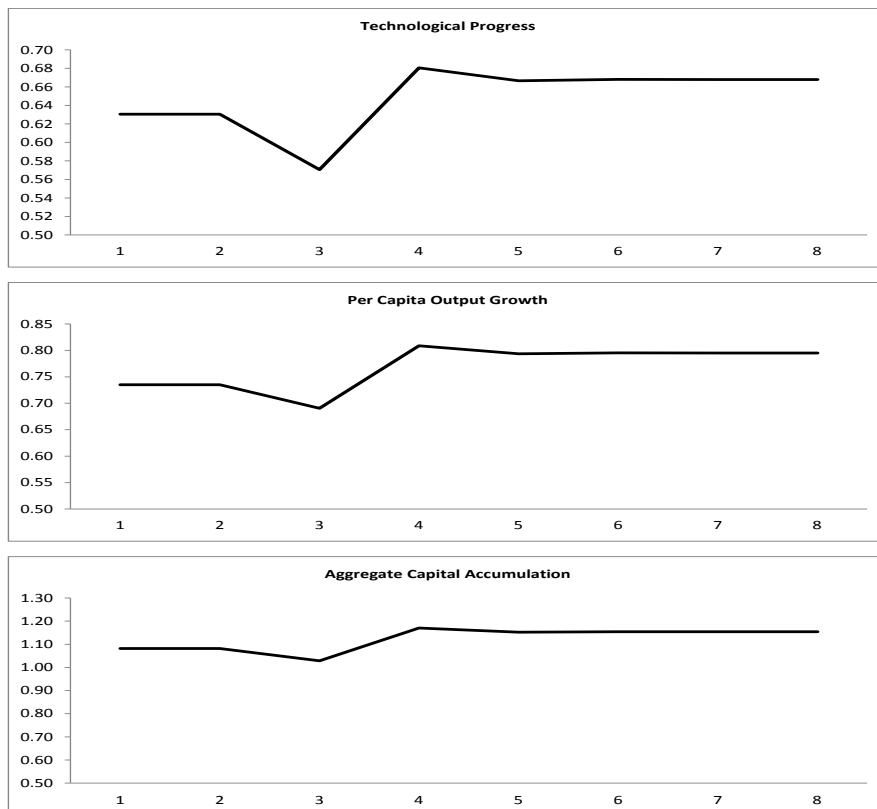
Table 2: Parameter values for simulation

Parameter	Value	Justification
β	0.3	implies a yearly discount rate of 5%
α	0.33	common in growth literature; see for example Jones (1995a)
δ	10	free to choose; parameter changes the magnitude of exogenous shocks during transition
ξ	10	ξ and ϕ imply g_y consistent with
ϕ	0.5	World Bank (2012) data for the United States
τ	0.0819	implied by World Bank (2012) data for the United States
ψ	0.091	common in human capital calibrations; see for example Psacharopoulos (1994) and Bloom and Canning (2005)
n	1.2	implies population growth of 0.7%

The results of doing so are depicted in figure 1. We assume that the economy initially moves along the balanced growth path. At generation 3 a 1 percentage point increase in public education expenditures as a fraction of GDP occurs. Afterwards the behavior of the economy is traced for another five generations, i.e., for 125 years.

We see that the effect of an increase in public educational investments at impact is such that labor is drawn away from the R&D sector into the education sector which slows down technological progress, per capita GDP growth and aggregate capital accumulation for one generation (this reflects the “near term costs” of education). In the subsequent generation, when the better educated workforce enters the labor market, the growth rate of technology, per capita GDP and the aggregate capital stock peak. This is due to an upward level shift of aggregate human capital *and* to faster growth

Figure 1: Simulation of an increase in public education expenditures



Time is displayed on the x-axis and growth is displayed on the y-axis. The economy initially moves along the balanced growth path. At generation three 3 is a 1 percentage point increase in public education expenditures as a fraction of GDP. Afterwards the economy is traced for another five generations, i.e., for 125 years.

of individual human capital. We are familiar with this transition behavior of semi-endogenous growth models in response to level shifts for example in the population size (cf. Trimborn et al., 2008). Afterwards the growth rates of technology, per capita GDP and aggregate capital converge to their new balanced growth path levels being higher than before the increase in educational investments. This is consistent with the claim expressed in proposition 2.

4 Discussion

We set up an R&D based economic growth model and extend it to allow for a *public* education sector. This allows us to

- generalize the R&D based growth literature to take into account an empirically important determinant of human capital accumulation and economic development (cf. Mankiw et al., 1992; Barro and Lee, 1994; Sachs and Warner, 1995; Hall and Jones, 1999; Bils and Klenow, 2000; Lutz et al., 2008). We show that the long-run growth rate of the economy is not only affected by technological progress (being itself driven by population growth and human capital investments) but is further enhanced by sustained increases in the skills of the labor force showing the multidimensional importance of education. Consequently, we present a potentially important mechanism that is able to bridge the gap between growth models relying solely on human capital accumulation like Lucas (1988) and the R&D based growth literature that is different from the mechanism present in Dalgaard and Kreiner (2001) and Strulik et al. (2011).
- show that the long-run growth rates of technology and per capita output are sensitive to changes in governmental education policies. Therefore we challenge a property of early semi-endogenous growth models in the vein of Jones (1995a), Kortum (1997) and Segerström (1998) in favor of later scale-free growth models in the spirit of Dinopoulos and Thompson (1998), Peretto (1998), Young (1998) and Howitt (1999).
- show that increases in population growth might harm long-run economic growth perspectives in case that the education sector of an economy is well developed. This primarily applies to industrialized countries in the second half of the twentieth century and therefore has the potential to explain the negative correlation between economic growth and population growth found in empirical studies for this time frame (cf. Brander and Dowrick, 1994; Kelley and Schmidt, 1995; Ahtuv, 2001; Bernanke and Gürkaynak, 2001). Nevertheless, our model is also consistent with historical evidence prior to the twentieth century: public educational sectors were less developed then and therefore our framework supports a positive correlation between economic growth

and population growth consistent with the findings of Kremer (1993).

From a policy perspective, our results imply that educational investments are very important to foster long-run economic development. However, there might be short-run costs associated with the implementation of growth promoting educational reforms. This essentially pins down to the trade-off between benefiting future generations at the expense of currently tax paying adults.

As already indicated, some aspects of the results in our paper have been shown within other frameworks. In particular, the notion that long-run economic growth is not completely driven by exogenous forces was the main reason for integrating horizontal and vertical innovations to remove the scale effect in otherwise endogenous growth models (cf. Dinopoulos and Thompson, 1998; Peretto, 1998; Young, 1998; Howitt, 1999). Moreover, *private* educational investments represent a main driving force behind long-run economic development in Dalgaard and Kreiner (2001) and Strulik et al. (2011). However, we are confident that our paper i) represents a consistent framework for analyzing these issues and their interrelations simultaneously, ii) sheds some light on the notion and importance of public education and especially the connection between years of schooling, teacher-pupil ratios and population growth, and iii) allows for a fairly general dependence between population growth and economic prosperity being consistent with the empirical evidence for modern times as well as with historical data.

We also acknowledge that our framework is highly stylized and some important issues cannot be treated within its realms. Possible extensions might therefore reveal other aspects of the connection between economic growth, education and demography. For example, the population growth rate and private educational investments could be endogenized along the lines of Strulik et al. (2011) to analyze potential feedback effects between public and privately financed education, fertility and the pupil-teacher ratio. In particular, this could prove to be a useful framework for analyzing the extent to which public and private education complemented one another in the course of the industrial revolution (cf. Mokyr, 2005; Galor, 2011).

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References

- Acemoglu, D. (2009). *Introduction to Modern Economic Growth*. Princeton University Press.
- Aghion, P. and Howitt, P. (1992). A model of growth through creative destruction. *Econometrica*, Vol. 60(No. 2):323–351.
- Ahituv, A. (2001). Be fruitful or multiply: On the interplay between fertility and economic development. *Journal of Population Economics*, Vol. 14:51–71.
- Barro, R. and Lee, J.-W. (1994). Sources of economic growth. *Carnegie-Rochester Conference Series on Public Policy.*, Vol. 40:1–57.
- Barro, R. J. and Lee, J.-W. (2010). A New Data Set of Educational Attainment in the World, 1950-2010. NBER Working Paper No. 15902.
- Becker, G. S. (1993). *A Treatise on the Family*. Harvard University Press.
- Bernanke, B. S. and Gürkaynak, R. S. (2001). Is Growth Exogenous? Taking Mankiw, Romer and Weil Seriously. NBER Working Papers 8365.
- Bils, M. and Klenow, P. J. (2000). Does Schooling Cause Growth? *American Economic Review*, Vol. 90(No. 5):1160–1183.
- Bloom, D. E. and Canning, D. (2005). Health and Economic Growth: Reconciling the Micro and Macro Evidence. Center on Democracy, Development and the Rule of Law Working Papers.
- Brander, J. A. and Dowrick, S. (1994). The role of fertility and population in economic growth. *Journal of Population Economics*, Vol. 7(No. 1):1–25.

- Caselli, F. (2005). *Handbook of economic growth*, volume 1, chapter Accounting for cross-country income differences, pages 679–741. Elsevier.
- Dalgaard, C. and Kreiner, C. (2001). Is declining productivity inevitable? *Journal of Economic Growth*, Vol. 6(No. 3):187–203.
- Diamond, P. A. (1965). National debt in a neoclassical growth model. *American Economic Review*, Vol. 55(No. 5):1126–1150.
- Diks, C., Hommes, C., Panchenko, V., and Weide, R. (2008). E&F Chaos: a user friendly software package for nonlinear economic dynamics. *Computational Economics*, Vol. 32(No. 1):221–244.
- Dinopoulos, E. and Thompson, P. (1998). Schumpeterian growth without scale effects. *Journal of Economic Growth*, Vol. 3:313–335.
- Dixit, A. K. and Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity. *American Economic Review*, Vol. 67(No. 3):297–308.
- Durlauf, S. N., Johnson, P. A., and Temple, J. R. (2005). *Handbook of Economic Growth, Volume 1A.*, chapter Growth Econometrics, pages 555–677.
- Galor, O. (2005). *Handbook of Economic Growth*, chapter From Stagnation to Growth: Unified Growth Theory, pages 171–293.
- Galor, O. (2011). *Unified Growth Theory*. Princeton University Press.
- Galor, O. and Weil, D. (2000). Population, technology, and growth: From malthusian stagnation to the demographic transition and beyond. *The American Economic Review*, Vol. 90(No. 4):806–828.
- Gersbach, H., Sorger, G., and Amon, C. (2009). Hierarchical Growth: Basic and Applied Research. Department of Economics, University of Vienna, Working Paper No: 0912.
- Grossman, G. M. and Helpman, E. (1991). Quality ladders in the theory of economic growth. *Review of Economic Studies*, Vol. 58(No. 1):43–61.
- Hall, R. and Jones, C. (1999). Why do Some Countries Produce So Much More Output Per Worker than Others? *Quarterly Journal of Economics*, Vol. 114(No. 1):83–116.

- Howitt, P. (1999). Steady endogenous growth with population and R&D inputs growing. *Journal of Political Economy*, Vol. 107(No. 4):715–730.
- Jones, C. I. (1995a). R&D-based models of economic growth. *Journal of Political Economy*, Vol. 103(No. 4):759–783.
- Jones, C. I. (1995b). Time Series Tests of Endogenous Growth Models. *The Quarterly Journal of Economics*, Vol. 110(No. 2):495–525.
- Kaldor, N. (1957). A model of economic growth. *The Economic Journal*, Vol. 67(No. 268):591–624.
- Kelley, A. C. and Schmidt, R. M. (1995). Aggregate population and economic growth correlations: the role of the components of demographic change. *Demography*, Vol 32(No. 4):543–555.
- Kortum, S. (1997). Research, patenting and technological change. *Econometrica*, Vol. 65(No. 6):1389–1419.
- Kremer, M. (1993). Population Growth and Technological Change: One Million B.C. to 1990. *The Quarterly Journal of Economics*, Vol. 108(No. 3.):681–716.
- Lucas, R. E. (1988). On the mechanics of economic development. *Journal of Monetary Economics*, 22:3–42.
- Lutz, W., Cuaresma, J. C., and Sanderson, W. (2008). The demography of educational attainment and economic growth. *Science*, Vol. 319(No. 5866):1047–1048.
- Mankiw, N. G., Romer, D., and Weil, D. N. (1992). A contribution to the empirics of economic growth. *Quarterly Journal of Economics*, Vol. 107(No. 2):407–437.
- Mincer, J. (1974). *Schooling, Experience and Earnings*. New York, NBER Press.
- Mokyr, J. (2005). *Handbook of Economic Growth, Volume 1B*, chapter Long-term economic growth and the history of technology, pages 1113–1180. Elsevier.

- Peretto, P. F. (1998). Technological change and population growth. *Journal of Economic Growth*, Vol. 3(No. 4):283–311.
- Prettner, K., Bloom, D. E., and Strulik, H. (2012). Declining fertility and economic well-being: do education and health ride to the rescue? Program on the Global Demography of Aging Working Paper No. 84.
- Psacharopoulos, G. (1994). Returns to Investment in Education: A Global Update. *World Development*, Vol. 22(No. 9):1325–1343.
- Romer, P. (1990). Endogenous technological change. *Journal of Political Economy*, 98(No. 5):71–102.
- Sachs, J. D. and Warner, A. (1995). Economic reform and the process of global integration. *Brookings Papers on Economic Activity*, (No. 1):1–118.
- Segerström, P. S. (1998). Endogenous growth without scale effects. *American Economic Review*, Vol 88.(No. 5):1290–1310.
- Strulik, H. (2005). The role of human capital and population growth in R&D-based models of economic growth. *Review of International Economics*, Vol. 13(No. 1):129–145.
- Strulik, H., Prettner, K., and Prskawetz, A. (2011). R&D based growth in the post-modern era. Program on the Global Demography of Aging. Working Paper 74/2011.
- Trimborn, T., Koch, K.-J., and Steger, T. M. (2008). Multidimensional transitional dynamics: a simple numerical procedure. *Macroeconomic Dynamics*, Vol. 12:301–319.
- World Bank (2012). World Development Indicators & Global Development Finance Database. URL: <http://databank.worldbank.org/ddp/home.do?step=12&id=4&cno=2>.
- Young, A. (1998). Growth without scale effects. *Journal of Political Economy*, Vol. 106(No. 5):41–63.